First Exam for MTH 23

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Name: ______________________________

Instructions:

This exam contains 6 pages (including this cover page) and 5 questions. Each question is worth 20 points, and so the perfect score in this exam is 100 points. Check to see if any pages are missing. Enter your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use only the provided formulae sheet. You may *not* use your book or notes.

You are allowed to use a calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- You have to enter the answer of each question in the provided box or blank line. You have to circle your answer in the multiple choice questions.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or other work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the last page; clearly indicate when you have done this.

1. Consider the following set of data:

$$23 \quad 15 \quad 27 \quad 15 \quad 30$$

   (a) Find the median. **Answer:** The median is $\boxed{}$

   (b) Find the mode. **Answer:** The mode is $\boxed{}$

   (c) Find the sample mean. **Answer:** $\bar{x} = \boxed{}$

   (d) Find the sample standard deviation. **Answer:** $s = \boxed{}$

2. About 45% of those called for jury duty will find an excuse to avoid it. Suppose 5 people are randomly called for jury duty, and let r stand for the number of people that actually show up, i.e. they do *not* find an excuse to avoid it.

   (a) Using the appropriate table, fill in the following chart:

r	0	1	2	3	4	5
$P(r)$						

   (b) Find the expected value μ and the standard deviation σ of this probability distribution.

Answer: The expected value is $\mu = \boxed{}$

The standard deviation is $\sigma = \boxed{}$

   (c) Determine the probability that all 5 serve on jury duty.

Answer: The probability is $\boxed{}$

3. Consider the experiment of rolling two dice. The following table lists all possible outcomes.

1 6	2 6	3 6	4 6	5 6	6 6
1 5	2 5	3 5	4 5	5 5	6 5
1 4	2 4	3 4	4 4	5 4	6 4
1 3	2 3	3 3	4 3	5 3	6 3
1 2	2 2	3 2	4 2	5 2	6 2
1 1	2 1	3 1	4 1	5 1	6 1

Find the probability that the sum of the outcomes of the two dice is less or equal to 5.

Answer: The probability is $\boxed{}$

4. In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new car. The results are summarized in the table below:

	Sale	**No Sale**	**TOTAL**
Aggressive	270	310	580
Passive	416	164	580
TOTAL	686	474	1160

Compute the following probabilities:

(a) Find the probability that a customer selected at random bought a new car.

Answer: The probability is $\boxed{}$

(b) Find the probability that a customer selected at random will buy a new car given that an aggressive sales approach is used.

Answer: The probability is $\boxed{}$

(c) Find the probability that a customer selected at random will buy a car **and** that an aggressive sales approach was used.

Answer: The probability is $\boxed{}$

5. The National Infomercial Marketing Association conducted a survey among *buyers* of a particular product, and the results are shown in the table below, where x represent the number of times buyers of a product had watched a TV infomercial before purchasing the product, and $p(x)$ the probability that buyers purchase the product after watching it x times. Values of 5 or more for x were treated as 5.

x	$p(x)$
1	0.27
2	
3	0.08
4	0.09
5	0.15

(a) Complete the missing probability.

(b) What is the probability that buyers watch three or fewer infomercials before purchasing the product?

Answer: The probability is $\boxed{}$

(c) Compute the expected value of this distribution.

Answer: The expected value is $\mu = \boxed{}$

(d) Compute the standard deviation of this distribution.

Answer: The standard deviation is $\sigma = \boxed{}$

Useful Formulae

Mean: $\bar{x} = \dfrac{\sum x}{n}, \qquad \mu = \dfrac{\sum x}{N}$

Standard Deviation: $s = \sqrt{\dfrac{\sum x^2 - \frac{1}{n}\left(\sum x\right)^2}{n-1}}, \qquad \sigma = \sqrt{\dfrac{\sum x^2 - \frac{1}{N}\left(\sum x\right)^2}{N}}$

Discrete random variables:

$$\mu = \sum x\, p(x) \qquad \sigma = \sqrt{\sum (x-\mu)^2\, p(x)} \quad \boxed{= \sqrt{\left(\sum x^2\, p(x)\right) - \mu^2}}$$

Binomial Distribution

$$q = 1 - p \qquad P(r) = \binom{n}{r} p^r\, q^{n-r} \qquad \mu = n\,p \qquad \sigma = \sqrt{n\,p\,q}$$

$$n! = 1 \cdot 2 \cdot 3 \cdots n \qquad \binom{n}{r} = \frac{\overbrace{n(n-1)\cdots(n-r+1)}^{r \text{ factors}}}{r!}$$