# Fourth quiz

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**Directions:** Please write your answers in separate papers and staple all the papers together. This exam is due Wdnesdayy, May 6, at 4:00 PM.

1. A graph $G$ has six vertices with the following degrees: $5, 3, 3, 2, 4, 1$.
   (a) How many edges does $G$ have?
   (b) Draw two different such graphs.
2. Prove that in every simple graph there are at least two vertices with the same degree.
   **Hint.** You may want to use induction.
3. For each of the families of graphs $K_n$, $P_n$, $C_n$, $W_n$, and $Q_n$:
   (a) find a formula for the number of edges of the graph.
   (b) Determine whether the graph is 2colorable.
   (c) Determine whether the graph has an Euler path or an Euler Circuit.
   (d) Determine whether the graph has a Hamilton path or Hamilton Circuit.
   (e) Determine the chromatic number of the graph.

   Prove your answers.
4. Are the two graphs in Figure 1 isomorphic? How about the graphs in Figure 2? or in Figure 3? Prove your answers.

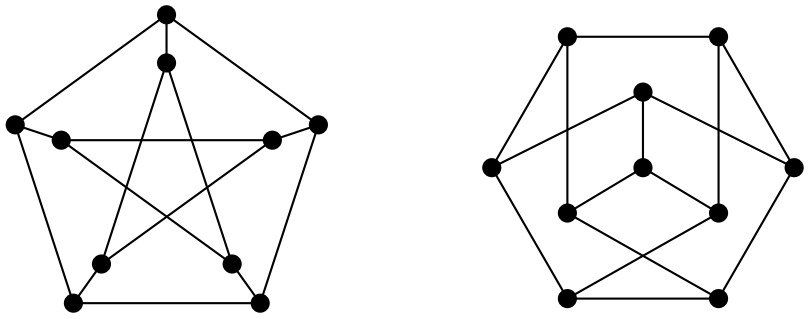

Figure 1: The first two graphs of Question 4

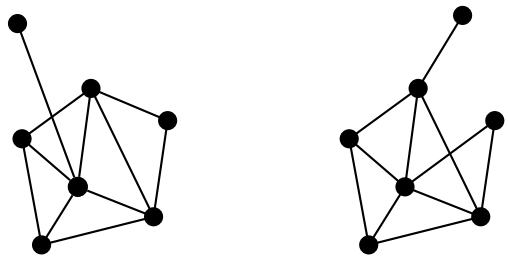

Figure 2: The middle two graphs of Question 4

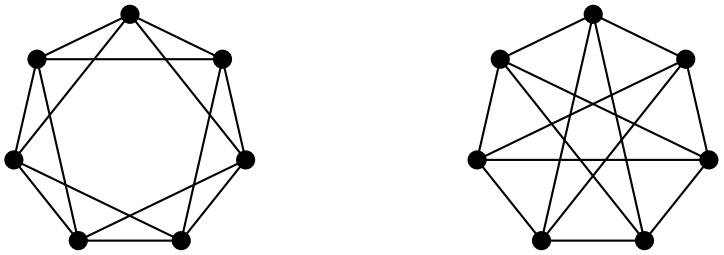

Figure 3: The last two graphs of Question 4

5. Count von Diamond has been murdered in his estate. The internationally known detective (and part time graph theorist) Inspector Clouseau has been called in to investigate. The butler claims that he saw the gardener enter the pool room (where the murder took place) and then, shortly after, leave that room by the same door. On the other hand, the gardener says that he cannot be the man that the butler saw because he entered the house, went through each door exactly once and then left the house. Inspector Clouseau checks the floor plan (see Figure 4) and within minutes declares the case solved. Who done it?

6. Find the shortest path from $a$ to $z$ in the weighted graph in Figure 5.

7. On a $3 \times 4$ chessboard there are three white knights on the top row and three black knights on the bottom row, as shown in the following picture. Using only legal moves, interchange the black and white knights in as few moves as you can.

   **Hint.** The solution is much easier if you work with the corresponding graph of legal moves.

8. Prove that the two graphs in Figure 7 are isomorphic.

9. Prove that the graph in the right side of Figure 7 is not Hamiltonian.

10. Prove that the $4 \times 4$ chessboard does not admit a knight's tour.

    **Hint.** Put the previous two questions together.

11. **Extra Credit:** Find a knight's tour on an $8 \times 8$ chessboard.

    **Hint.** There are many knight's tours on the standard chessboard. One way to proceed (invented by our old acquaintance Leonard Euler) is to first find an open knight's tour on a $4 \times 8$ board

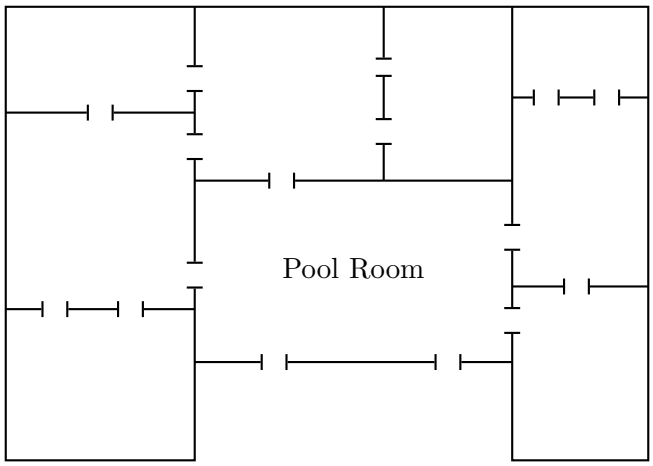


Figure 4: The floor of Von Diamond Estate

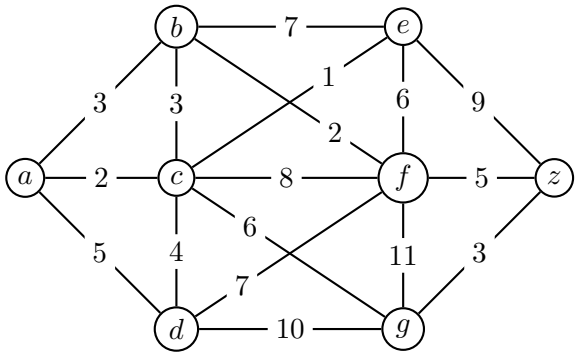


Figure 5: The weighted graph of Question 6

that starts at the top row and ends two squares to its left; to find a knight's tour on the $8 \times 8$ board then you just "glue" that open tour and its mirror image.

| | | |
|---|---|---|
| W | W | W |
| | | |
| | | |
| B | B | B |

Figure 6: The six knights of Question 7

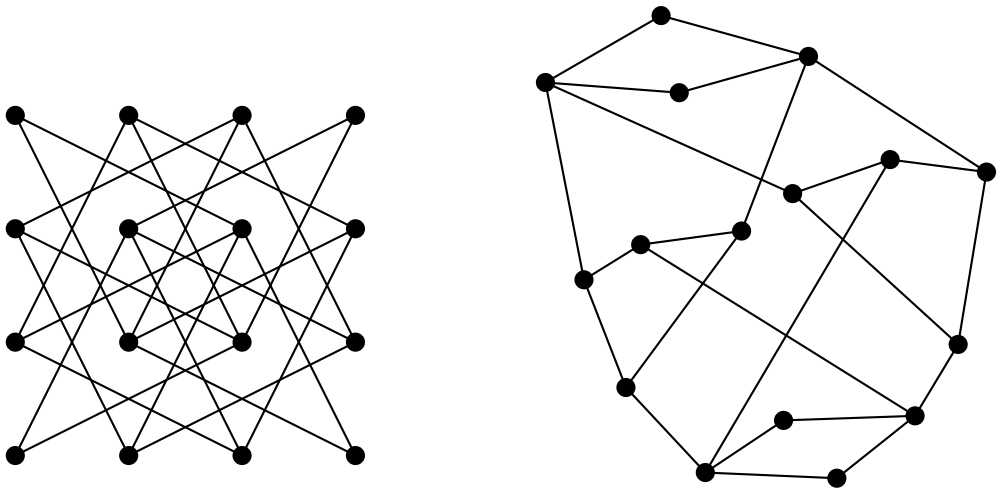

Figure 7: The graphs of Question 8