

First Quiz for CSI35

Nikos Apostolakis

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Directions: This quiz is due Wednesday February 18, at 4:00 PM.

1. Prove by induction that for all integers $n \geq 5$ we have:

$$\sum_{i=5}^n i = \frac{(n-4)(n+5)}{2}$$

2. Prove that for all $n \in \mathbb{N}$, 5 divides $n^5 - n$.
3. In Nevereverland chicken nuggets come in packages of 3 and 5. Prove that for any integer $n \geq 8$ a nevereverlander can combine packages to get a total of exactly n chicken nuggets.
4. Alice and Bob play a game by taking turns removing 1, 2 or 3 stones from a pile that initially has n stones. The person that removes the last stone wins the game. Alice plays always first.
 - (a) Prove by induction that if n is a multiple of 4 then Bob has a winning strategy.
 - (b) Prove that if n is not a multiple of 4 then Alice has a winning strategy.
5. **Extra Credit** In a party with at least two people, every person shakes hands with the people they know. Any two given people will either not shake hands or they will shake hands exactly once. Show that there will always be at least one pair of people who shake the same number of hands.