Second Quiz
The solutions

1. Solve the following equation: $3(2x-1) = 4x+3$

   Solution. We first expand in the left hand side to get:

   $$6x - 3 = 4x + 3$$

   Then we isolate the unknowns by transferring -3 to the right hand side and $4x$ to the left hand side. The equation becomes:

   $$6x - 4x = 3 + 3$$

   Which is equivalent to:

   $$2x = 6$$

   Now we can solve by dividing both sides by 2:

   $$x = 3$$

   □

2. Solve the following equation:

   $$\frac{2x+9}{5} + x = \frac{13-x}{10} - 10$$

   Solution. We first get rid of denominators by multiplying both sides with 10 (the L.C.D.). We get:

   $$\cancel{10}^{2} \cdot \frac{2x+9}{\cancel{5}^{1}} + 10 \cdot x = \cancel{10}^{1} \cdot \frac{13-x}{\cancel{10}^{1}} - 10 \cdot 10 \iff 2 \cdot (2x+9) + 10x = (13-x) - 100$$

   $$\iff 4x + 18 + 10x = 13 - x - 100$$
   $$\iff 14x + 18 = -x - 87$$
   $$\iff 14x + x = -87 - 18$$
   $$\iff 15x = -105$$
   $$\iff x = \frac{-105}{5}$$
   $$\iff x = -7$$

   □

3. Find the real number a if $x = \frac{3}{2}$ solves the following equation:

   $$ax - 5 = -2x + 1$$

Solution. If we substitute $x = \frac{3}{2}$ in the equation we get:

$$\begin{aligned} a\left(\frac{3}{2}\right) - 5 = -2\left(\frac{3}{2}\right) + 1 &\iff \frac{3}{2}a - 5 = -3 + 1 \\ &\iff \frac{3}{2}a - 5 = -2 \\ &\iff 2 \cdot \frac{3}{2}a - 2 \cdot 5 = -2 \cdot 2 \\ &\iff 3a - 10 = -4 \\ &\iff 3a = -4 + 10 \\ &\iff 3a = 6 \\ &\iff a = 2 \end{aligned}$$

Therefore a is 2. □

4. Consider the equation:

$$4(2x - 3) - 5x + 3 = -5(2 - x) - 2x + 7$$

Which of the following is true?

A. Only the number 0 is solution.

B. Only the number -6 is solution.

C. All real numbers are solutions.

D. There are no solutions.

Solution. we have:

$$\begin{aligned} 4(2x - 3) - 5x + 3 = -5(2 - x) - 2x + 7 &\iff 8x - 12 - 5x + 3 = -10 + 5x - 2x + 7 \\ &\iff 3x - 9 = 3x - 3 \\ &\iff -9 = -3 \end{aligned}$$

The last equation is a *contradiction* and therefore the initial equation which is equivalent to it has no solutions. So the correct answer is **D**. □

5. For a linear equation with one unknown both 0 and -7 are solutions. Which of the following must necessarily be true?

A. There are no other solutions.

B. -3.5 is also a solution.

C. We can't know all solutions.

D. This can't happen with a linear equation.

Solution. For a linear equation with one unknown there are only three, mutually exclusive, possibilities:

1. There is *exactly one* solution.
2. All real numbers are solutions.
3. There are no solutions.

Since the equation has two different solutions, case (1) and (3) cannot be true, and therefore case (2) is true: all real numbers are solutions. Since -3.5 is a real number it follows that it is a solution. Thus the correct answer is **B**. □