The Answers to the Second Exam

1. Evaluate: $-7+5(6-8)$

   Solution.

$$\begin{aligned} -7+5(6-8) &= -7+5(-2) \\ &= -7-10 \\ &= -17 \end{aligned}$$

□

2. Evaluate: $\sqrt{c^2-a^2}$, when $c=-10$ and $a=6$.

   Solution.

$$\begin{aligned} \sqrt{c^2-a^2} &= \sqrt{(-10)^2-(6)^2} \\ &= \sqrt{100-36} \\ &= \sqrt{64} \\ &= 8 \end{aligned}$$

□

3. Let $f(x)=x^2-5x+4$. Find $f(-3)$.

   Solution.

$$\begin{aligned} f(-3) &= (-3)^2-5(-3)+4 \\ &= 9-5(-3)+4 \\ &= 9+15+4 \\ &= 28 \end{aligned}$$

□

4. Translate into algebra and solve:

   10 **is** 10 **more than** 5 **times a number.**

   Solution. Let x be the unknown number. Then 10 more than 5 times the number is $5x+10$. So the given phrase translates to the equation:

$$10 = 5x+10$$

   Now we solve the equation:

$$\begin{aligned} 10 = 5x+10 &\iff 0 = 5x \\ &\iff 0 = x \end{aligned}$$

   Thus the unknown number is 0. □

5. Solve the equation: $2(x+5) = 3(x+8) - 6$

Solution. We have:

$$\begin{aligned} 2(x+5) = 3(x+8) - 6 &\iff 2x + 10 = 3x + 24 - 6 \\ &\iff 2x + 10 = 3x + 18 \\ &\iff 10 - 18 = 3x - 2x \\ &\iff -8 = x \end{aligned}$$

□

6. Solve for x: $3y = 5x + 4z$

Solution. We have

$$\begin{aligned} 3y = 5x + 4z &\iff 3y - 4z = 5x \\ &\iff \frac{3y - 4z}{5} = x \\ &\iff x = \frac{3y - 4z}{5} \end{aligned}$$

□

7. Evaluate. Give the answer in Scientific Notation.

$$\left(7.3 \times 10^{-3}\right) \times \left(5.0 \times 10^{7}\right)$$

Solution.

$$\begin{aligned} \left(7.3 \times 10^{-3}\right) \times \left(5.0 \times 10^{7}\right) &= (7.3 \times 5.0) \times \left(10^{-3} \times 10^{7}\right) \\ &= 36.5 \times 10^{4} \\ &= 3.65 \times 10^{5} \end{aligned}$$

□

8. Evaluate. Give the answer in Scientific Notation.

$$\frac{2.3 \times 10^{5}}{5.0 \times 10^{-9}}$$

Solution.

$$\begin{aligned} \frac{2.3 \times 10^{5}}{5.0 \times 10^{-9}} &= \frac{2.3}{5.0} \times \frac{10^{5}}{10^{-9}} \\ &= 0.46 \times 10^{14} \\ &= 4.6 \times 10^{13} \end{aligned}$$

□

9. Find the point of intersection of the lines with equations $y = 3x - 1$ and $5x - 2y = 3$.

Solution. We need to solve the system of the two equations. Substituting the expression for y from the first equation into the second we have:

$$\begin{aligned} 5x - 2(3x - 1) = 3 &\iff 5x - 6x + 2 = 3 \\ &\iff -x = 1 \\ &\iff x = -1 \end{aligned}$$

We now substitute the value of x in the first equation to find the value of y:

$$\begin{aligned} y = 3(-1) - 1 &\iff y = -3 - 1 \\ &\iff y = -4 \end{aligned}$$

So the two lines intersect at the point with coordinates $(-1, -4)$. □

10. Find the slope and the two intercepts of the line with equation $-7x + 3y = -42$.

Solution. We put the equation in slope-intercept form by solving for y:

$$\begin{aligned} -7x + 3y = -42 &\iff 3y = 7x - 42 \\ &\iff y = \frac{7x - 42}{3} \\ &\iff y = \frac{7}{3}x - 14 \end{aligned}$$

So the slope of the line is $\dfrac{7}{3}$ and its y-intercept is at $(0, -14)$.

To find the x-intercept we substitute $y = 0$ in the equation and solve for x:

$$\begin{aligned} -7x + 3 \cdot 0 = -42 &\iff -7x = -42 \\ &\iff x = 6 \end{aligned}$$

So the x intercept is at $(6, 0)$. □

11. Sketch the graph of $4x + 3y = 12$. Show the x and y intercepts.

Solution. We need two points to graph the line. Since we are also asked for the two intercepts we start by finding those.

The x-intercept is the point of the line for which $y = 0$.

$$\begin{aligned} 4x + 3 \cdot 0 = 12 &\iff 4x = 12 \\ &\iff x = 3 \end{aligned}$$

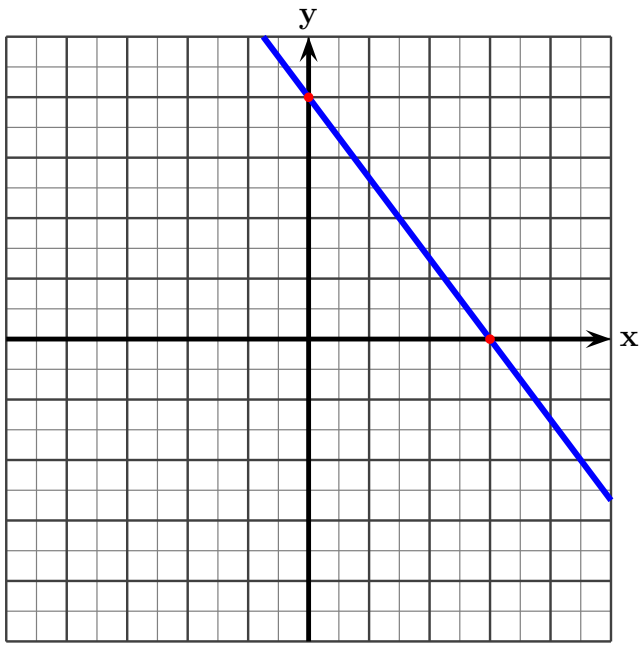
So the x-intercept is at $(3, 0)$.

The y-intercept is the point of the line for which $x = 0$.

$$4 \cdot 0 + 3y = 12 \iff 3y = 12$$
$$\iff y = 4$$

So the y-intercept is at $(0, 4)$.

So we have the following graph:



□

12. Solve the following system: $\begin{cases} 3x - 5y &= -1 \\ 4x + 2y &= 16 \end{cases}$

Solution. We begin by eliminating x: we multiply the first equation with 4 and the second with -3.

$$\begin{cases} 12x - 20y &= -4 \\ -12x - 6y &= -48 \end{cases}$$

Adding the two equations gives:

$$-26y = -52 \iff y = 2$$

Substituting the value of y in the first equation gives:

$$3x - 5 \cdot 2 = -1 \iff 3x = 9 \iff x = 3$$

So the solution is $(3, 2)$. □

13. Solve the following system: $\begin{cases} 2x + 3y &= -3 \\ 4x + 6y &= -6 \end{cases}$

Solution. We multiply the first equation with 4 and the second with -2. We get:

$$\begin{cases} 8x + 12y &= -12 \\ -8x - 12y &= 12 \end{cases}$$

Then we add up the two equations and get

$$0 = 0$$

So the system is equivalent to one of its equations. So all solutions to $2x + 3y = -3$ are solutions to the system. □

14. Simplify. Give your answer using positive exponents only. $(4x^{-4}y^3z^5)^2(-2x^{-2}y^4z^{-2})^{-3}$

Solution.

$$\begin{aligned}(4x^{-4}y^3z^5)^2(-2x^{-2}y^4z^{-2})^{-3} &= 4^2x^{-8}y^6z^{10}(-2)^{-3}x^6y^{-12}z^6 \\ &= 16x^{-2}y^{-6}z^{16}\left(-\frac{1}{2^3}\right) \\ &= -\frac{2z^{16}}{x^2y^6}\end{aligned}$$

□

15. Simplify. Give your answer using positive exponents only.

$$\frac{x^2\left(y^2w^{-2}\right)^2}{x^{-3}y^3w^{-6}}$$

Solution.

$$\begin{aligned}\frac{x^2\left(y^2w^{-2}\right)^2}{x^{-3}y^3w^{-6}} &= \frac{x^2y^4w^{-4}}{x^{-3}y^3w^{-6}} \\ &= x^{2-(-3)}y^{4-3}w^{-4-(-6)} \\ &= x^5yw^2\end{aligned}$$

□

16. Simplify: $(-x^2 + 4x - 7) - (8x^2 + 3x - 2)$

Solution.

$$\begin{aligned}(-x^2 + 4x - 7) - (8x^2 + 3x - 2) &= -x^2 + 4x - 7 - 8x^2 - 3x + 2 \\ &= -9x^2 + x - 5\end{aligned}$$

□

17. Expand and simplify: $(2x-5)(3x^2-5x+7)$

Solution.

$$\begin{aligned}(2x-5)(3x^2-5x+7) &= 6x^3-10x^2+14x-15x^2+25x-35\\ &= 6x^3-25x^2+39x-35\end{aligned}$$

□

18. Expand and simplify: $(a+2)^3$

Solution.

$$\begin{aligned}(a+2)^3 &= (a+2)(a+2)^2\\ &= (a+2)(a^2+4a+4)\\ &= a^3+4a^2+4a+2a^2+8a+8\\ &= a^3+6a^2+12a+8\end{aligned}$$

□

19. Simplify: $\dfrac{10a^5b^3-4a^3b^2+6a^4b^6+8ab^2}{2ab^2}$

Solution. We have:

$$\begin{aligned}\frac{10a^5b^3-4a^3b^2+6a^4b^6+8ab^2}{2ab^2} &= \frac{10a^5b^3}{2ab^2}-\frac{4a^3b^2}{2ab^2}+\frac{6a^4b^6}{2ab^2}+\frac{8ab^2}{2ab^2}\\ &= 5a^4b-2a^2+3a^3b^4+4\end{aligned}$$

□

20. Simplify: $\dfrac{(2x-3)^2+24x}{(2x+3)^2}$

Solution.

$$\begin{aligned}\frac{(2x-3)^2+24x}{(2x+3)^2} &= \frac{4x^2-12x+9+24x}{4x^2+12x+9}\\ &= \frac{4x^2+12x+9}{4x^2+12x+9}\\ &= 1\end{aligned}$$

□