

Fourth set of Homework

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Please note: You should fully justify your answers.

Division of polynomials, Factors, roots

1. Solve the following quadratic equations:

(a) $x^2 - 6x + 13 = 0$

(b) $4x^2 - 4x - 2 = 0$

(c) $2x^2 + 9x - 35 = 0$

2. What is the remainder of this division: $\frac{x^{100} + 45x^{49} + 2}{x + 1}$

3. Given that $x = 5$ is a solution to the following equation

$$x^3 - 7x^2 + 15x - 25 = 0$$

find all solutions.

4. Given that $x = -2$ is a solution to the following equation:

$$x^4 - 5x^3 + x^2 + 5x - 50 = 0$$

solve the equation completely.

5. $x = 3$ is a solution to the equation

$$x^3 - 9x^2 + 27x - 27 = 0$$

Solve the equation completely.

6. One of the numbers 1, -2, 3, 4 is a solution to the equation

$$x^3 - 3x^2 - 10x + 24 = 0$$

Solve the equation.

7. Given that $2 + 5i$ is a solution of the equation:

$$x^4 - 3x^3 + 19x^2 + 53x - 174 = 0$$

solve this equation completely.

8. Find a cubic polynomial with zeros at $x = -1$, $x = 3$ and $x = 2$.

9. Find a fourth degree polynomial with real coefficients and zeros at $x = 3i$, $x = 2$, and $x = 0$.

10. **Extra Credit:** Given that $1 - \sqrt{5}$ is a solution to the equation

$$x^4 + 3x^3 - 8x^2 - 32x - 24 = 0$$

solve the equation completely.