Third set of Homework

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Please note: You should fully justify your answers.

1 The difference quotient and one-to-one functions

1. Find a formula difference quotient of the following functions:
   (a) $f(x) = 7$
   (b) $g(x) = -3x + 1$
   (c) $f(x) = x^2 - 3x$
   (d) $g(x) = 3x^2 - 5x + 7$
   (e) $f(x) = x^3 + 5x$
   (f) $g(x) = x^3 - 2x^2 + 3x + 4$

2. **Extra Credit** For each of the functions of the previous question use the difference quotients you computed to decide whether the function is 1-1.

2 Inverse of functions

1. Verify that the following are pairs of inverse functions:
   (a) $f(x) = 3x - \frac{1}{2}$, $g(x) = \frac{2x+1}{6}$
   (b) $f(x) = \sqrt[3]{x+5}$, $g(x) = x^3 - 5$
   (c) $g(x) = \frac{3x-2}{2x+3}$, $h(x) = -\frac{3x+2}{2y-3}$
   (d) $h(x) = x^2 - 3$ with domain $[0, \infty)$, $g(x) = \sqrt{x+3}$
   (e) $f(x) = 2 - \sqrt{x+7}$, $h(x) = x^2 - 4x - 3$ with domain $(-\infty, 2]$
   (f) $f(x) = \log_{10}(3x - 5)$, $g(x) = \frac{10^x + 5}{3}$

2. Are the functions $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses?

3. A function is called an *involution* if it is its own inverse. In other words, a function f is an involution if for all x in the domain of f, we have that $(f \circ f)(x) = x$. Show that the following functions are involutions:
   (a) $f(x) = \frac{1}{x}$
   (b) $g(x) = \sqrt{16 - x^2}$ with domain $[0, 4]$
   (c) $f(x) = \frac{2x-3}{4x-2}$

4. **Extra Credit** Is the function $f(x) = \sqrt{16 - x^2}$ with domain $[-4, 0]$ an involution? Justify your answer.

5. **Extra Credit** Is it possible to restrict the domain of the function $f(x) = 42$ so that it becomes an involution?

6. For the following pair of functions determine the compositions $f \circ g$ and $g \circ f$. In each case you should give the domain as well as the formula.
   (a) $f(x) = 3x - 1$, $g(x) = 2x + 3$
   (b) $f(x) = x - 2$, $g(x) = 5x^2 - 2$
   (c) $f(x) = x^2 - 3x + 5$, $g(x) = 2x - 3$
   (d) $f(x) = -2x^2 + x - 4$, $g(x) = x^2 + 1$
   (e) $f(x) = x^2 - 4$, $g(x) = \sqrt{x + 3}$
   (f) $f(x) = \dfrac{2x - 1}{5x + 3}$, $g(x) = \dfrac{x + 2}{x + 1}$
   (g) $f(x) = \sqrt{x - 3}$, $g(x) = 3 - x$
   (h) $f(x) = \dfrac{2x}{x^2 - 4}$, $g(x) = \frac{1}{x} - 2$
   (i) $f(x) = x^2 + 4$, $g(x) = \sqrt{3 - x}$
   (j) $f(x) = x$, $g(x) = 2^{\sin x}$
   (k) $f(x) = -x$, $g(x) = \sqrt{x}$
   (l) $f(x) = 3$, $g(x) = x^2 - 5x + 5$
   (m) $f(x) = x^2 + 3x - 7$, $g(x) = \sqrt{x - 1} + 1$
   (n) $f(x) = \cos 3x$, $g(x) = x^2 - 1$
   (o) $f(x) = \log_2 x$, $g(x) = -\sqrt{x + 3}$

7. If $f(0) = -4$ and $g(-4) = 6$ what is $(g \circ f)(0)$?

8. The graph of the functions f and g are shown in Figure 1. Find the following values:
   (a) $(f \circ g)(0)$
   (b) $(f \circ g)(-2)$
   (c) $(g \circ f)(1)$
   (d) $(g \circ f)(-1)$
   (e) $(g \circ f)(-4)$

9. Let $l(x) = x + 3$. For each of the following functions f,
   (a) find $f \circ l$, $l \circ f$
   (b) graph $y = f(x)$, $y = (f \circ l)(x)$, $(l \circ f)(x)$ on the same grid.

   1. $f(x) = x^2$
   2. $f(x) = -x^2$
   3. $f(x) = x^3$
   4. $f(x) = |x|$

10. Repeat the previous exercise with $l(x) = x - 2$

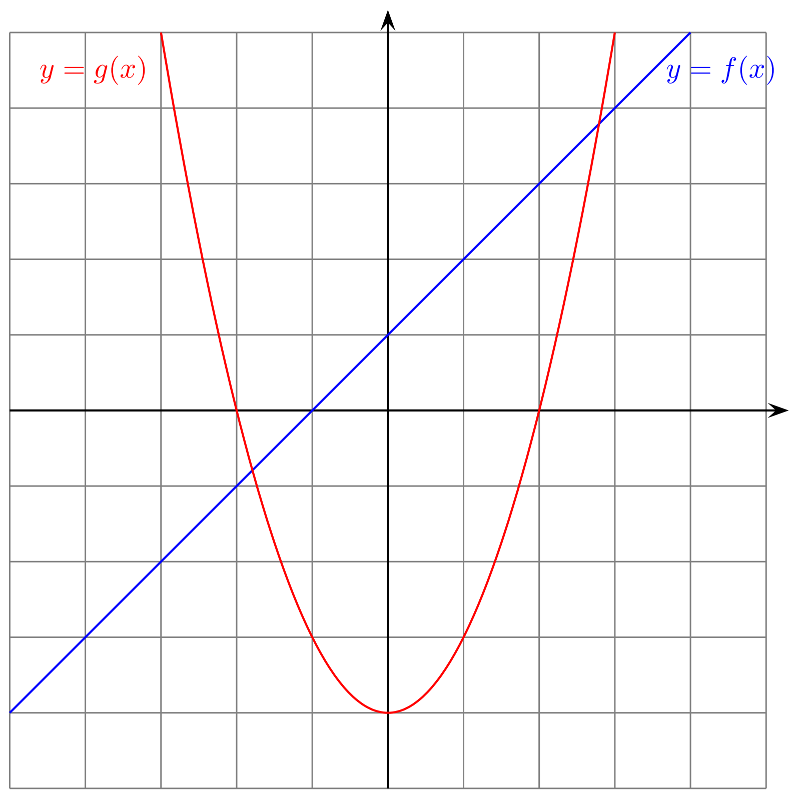

Figure 1: Two functions