Second set of Homework

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Please note: You should fully justify your answers.

1 Inverse of functions

1. Which of the following functions are one-to-one?
   (a) $f(x) = 42$
   (b) $g(x) = -2x + 5$
   (c) $h(x) = x^2 - 3$
   (d) $f(x) = x^2 + 1$ with domain $[0, \infty)$.
   (e) $g(x) = x^3$
   (f) $h(x) = x^3 - 8$
   (g) $g(x) = \sqrt{x+1}$
   (h) $f(x) = \sqrt{1 - x^2}$
   (i) $f(x) = (x - 2)^3$
   (j) $h(x) = \dfrac{3}{2x - 4}$
   (k) $g(x) = \dfrac{3x + 6}{x + 1}$
   (l) $f(x) = \dfrac{2x - 3}{5x - 2}$
   (m) $f(x) = x^2 - 3x + 2$
   (n) $g(x) = x^2 + 2x + 4$
   (o) $h(x) = \sin x$
   (p) $f(x) = 2^{x+1}$
   (q) $g(x) = \log_2(x - 1)$
   (r) $f(x) = (x - 1)(x - 2)(x - 3)$.
2. For each of the functions of the above exercise except the last,
   (a) if the function is one-to-one find the inverse function.
   (b) if the function is not one-to-one then find a maximal interval so that restricting the function to that interval makes it one-to-one.
3. Give an example of a relation that is not a function but its inverse is a function.
4. Prove that if a function is one-to-one then its inverse function is also one-to-one.

The exercises in this page are **Extra credit**

Recall the following from the textbook:

- A function is called *even*, if the following two conditions are satisfied:
    1. The domain of the function is “symmetric around 0”, that is if a number a is in the domain then $-a$ is also in the domain.
    2. For all a in the domain, $f(a) = f(-a)$.
- A function is called *odd*, if the following two conditions are satisfied:
    1. The domain of the function is “symmetric around 0”, that is if a number a is in the domain then $-a$ is also in the domain.
    2. For all a in the domain, $f(a) = -f(-a)$.

To find out if a function is even, odd, or neither:

1. Find $f(-x)$ and simplify it (remember that, for example, $(-x)^2 = x^2$, and $(-x)^3 = -x^3$).
2. Compare $f(-x)$ with $f(x)$:

- If $f(-x) = f(x)$ then the function is even.
- If $f(-x) = -f(x)$ then the function is odd.
- If none of the above, the function is neither (most functions are neither).

5. Use the above procedure to determine whether the functions below are even, odd or neither:
    (a) $f(x) = x^2 + 4$.
    (b) $g(x) = \dfrac{x^3}{x^2 + 4}$.
    (c) $h(x) = x^2 + x$.
6. Can you give an example of an even function that is one-to-one?
7. Can you give an example of an odd function that is one-to-one?