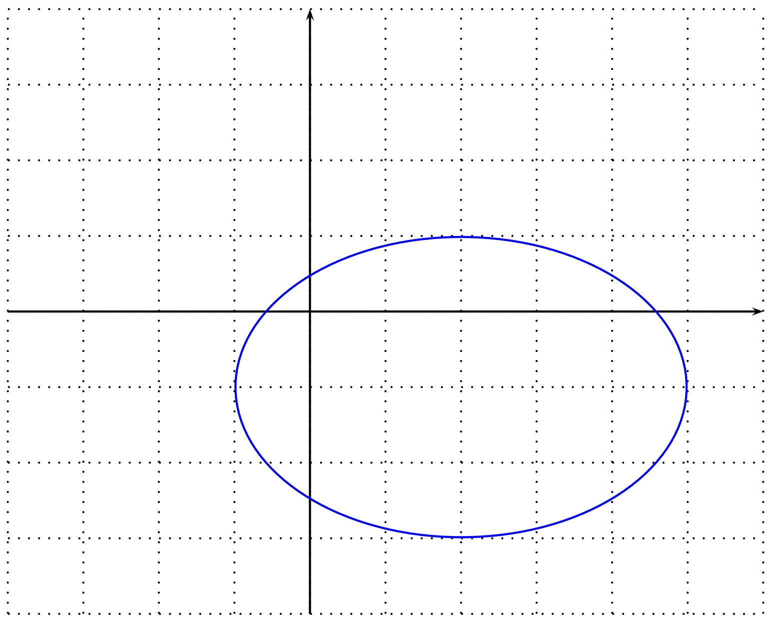
Second Exam

Take home

Due: April 28

1. The graph of the ellipse
$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$$
is shown bellow:



   (a) Explain why this is not the graph of a function.

   (b) How can we restrict the range so that we obtain a function?

   (c) Is the function you obtained in the previous step one-to-one? If not how can you restrict the domain so that it becomes one-to-one?

2. Let $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{3}{x}$. Find $f \circ g$. Your answer should include the domain as well as the formula.

3. Prove that $f(x) = \dfrac{2x-5}{3x+2}$ and $g(x) = -\dfrac{2x+5}{3x-2}$ are a pair of inverse functions.

4. Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 1$. Are f and g a pair of inverse functions? Justify your answer.

5. Sketch a graph of each of the following functions. The graph should correctly reflect end behavior, x and y intercepts, and possible asymptotes:

   (a) $f(x) = -x^3 + 4x^2 + 11x - 30$

(b) $g(x) = \dfrac{2x+4}{x^2-3x-18}$

6. Find the domain of each of the following functions:

   (a) $f(x) = \sqrt{\dfrac{x+3}{x-4}}$

   (b) $g(x) = ln(x^4 + 2x^3 - 16x^2 - 2x + 15)$

7. Let $f(x) = e^{3x-5}$.

   (a) Find the inverse function f^{-1}.

   (b) Sketch both functions on the same coordinate system.

8. Suppose $\log_5 a = 4$, $\log_5 b = 3$ and $\log_5 c = -2$. Evaluate the following expression:

$$\log_5\left(\frac{25b^3\sqrt{a}}{c^5}\right)$$

9. Solve $\quad \log_2(2x+8) - \log_2(x-3) = 4$

10. Sketch two full cycles of the graph of $y = 3\sin 2x$.

11. **Extra Credit:** Given that the remainder of the division

$$\frac{x^4 - 2x^3 + 5x^2 + 10x - 20}{x - \sqrt{5}}$$

is 30, solve the following equation:

$$x^4 - 2x^3 + 5x^2 + 10x - 50 = 0$$