Using the quadratic formula

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February 2, 2011

Recall 1. Recall that last time we derived the *quadratic formula:* The solution to the quadratic equation

$$ax^2 + bx + c = 0$$

where $a \neq 0$ is given by the formula:

$$x = \frac{-b \pm \sqrt{D}}{2a} \tag{1}$$

where,

$$D = b^2 - 4ac \tag{2}$$

is the *discriminant* of the equation.

1 The significance of the discriminant

The nature of the solutions to a given quadratic equation can be determined by its discriminant, see Equation (2). Indeed, we have that

- if $D > 0$ then we have two distinct *real* solutions.
- if $D = 0$ then we have one *double* real solution, $x = -\dfrac{b}{2a}$
- if $D < 0$ then we have *no real solutions*: both solutions are complex numbers.

Example 1. Find the real number b so that the equation

$$x^2 + bx + 5 = 0$$

has a double solution.

Answer. In order for the equation to have a double solution we need the discriminant to be 0. But $D = b^2 - 4 \cdot 5 = b^2 - 20$. So we need

$$b^2 - 20 = 0$$

Which gives two possible values: $b = \pm 2\sqrt{5}$. □

2 Exercises

1. Find the real number b so that the following equation:

$$9x^2 + bx + 25 = 0$$

   has exactly one (double) real solution.

2. For which real numbers a the equation $ax^2 - 4x + 7 = 0$ has real solutions?

3. For which real numbers c the equation:

$$3x^2 - 5x + c = 0$$

   has no real solutions?

4. Find the real number a if the equation: $ax^2 - 12x + 2a + 1 = 0$ has a double solution.