**Fourth set of Homework**

Nikos Apostolakis

**Please note:** You should fully justify your answers.

# 1 Some geometric problems

1. In the coordinate plane, choose any three points that do not lie in the same line and connect them using straight line segments to make a triangle. Then find the midpoints of two of the sides and connect them with a straight line segment. Verify that this new segment is parallel to the other side of the triangle.

2. In the coordinate plane, choose any four points that are the vertices of a quadrilateral. Connect the midpoints of consecutive sides by straight line segments to make a new quadrilateral. Verify that this new quadrilateral is a parallelogram.

3. Find an equation for the geometric locus of points that are equidistant from the points $(-1, -3)$ and $(2, -5)$. Verify that this is the perpendicular bisector of the segment defined by these two points.

4. Find an equation for the geometric locus of points that are equidistant from the two axes. Can you find the graph of this equation?

5. Find an equation for the geometric locus of points that are at distance 1 from the origin of the coordinate system (that is from $(0, 0)$).

6. Find an equation for the locus of points that are at distance 6 from the $y$–axis.

7. Find an equation for the set of points that are equidistant from the $x$–axis and the point $(0, 4)$. Can you graph this equation?

8. Find the distance of the point $(2, -3)$ from the line with equation $x - 2y = 3$.

9. Find the distance of the point $(-3, -5)$ from the line $x + y = -4$.

# 2 Some Extra Credit Problems

1. Prove by algebra that the midpoints of the sides of any quadrilateral are the vertices of a parallelogram.

   **Hint.** Choose a suitable coordinate system so that two of the points have easy coordinates.

2. What can you say about the "midpoint-parallelogram" of the previous question when the original quadrilateral is a rectangle? How about when the original parallelogram is a square?

3. Prove by algebra that the medians of a triangle pass through the same point.