

## Fourth Exam

Take home

**Due:** Thursday, April 28

- The length of one leg of a right triangle is one unit more than the length of the other leg. The length of the hypotenuse is  $\sqrt{41}$  units.
  - Find the lengths of the two legs.
  - Find the measure of the two acute angles of the triangle.
- The angle of elevation of the top of a building taken 300 feet from the base of the building is  $22^\circ$ . Find the height of the building to the nearest foot.
- Simplify:  $5\sqrt{52} - 3\sqrt{60} + 2\sqrt{13} + 3\sqrt{135}$
- Simplify:  $\frac{2\sqrt{5} - 5\sqrt{2}}{\sqrt{10} - 2}$
- Simplify assuming all variables represent positive numbers. The answer should contain only positive integers as exponents.

$$\left(\frac{27x^{15}y^{-\frac{21}{2}}}{8z^{-\frac{3}{2}}}\right)^{-\frac{1}{3}}$$

- Solve:  $\sqrt{x-5} - \sqrt{x-1} = 3$
- Simplify. Express your answer in the form  $a + bi$  where  $a$  and  $b$  are real numbers.

$$\frac{(3 + 2i)^2}{12 - 5i}$$

- Divide:  $\frac{2x^2 + 4x - 30}{-x^2 + 11x - 24} \div \frac{-x^2 + x + 30}{x^2 - 14x + 48}$ . Simplify the result as much as possible.
- Simplify:  $\frac{\frac{2}{b-3} - \frac{3}{b+2}}{1 - \frac{11b+7}{b^2-b-6}}$
- Solve:  $\frac{1}{x-2} + 3 = \frac{x}{x+3} + \frac{3x^2 + 9x - 25}{x^2 + x - 6}$
- Find all solutions  $\theta$ , with  $0^\circ \leq \theta < 360^\circ$ :

$$2 \sin \theta = -1$$

12. Find all angles  $\theta$  with  $0^\circ \leq \theta < 360^\circ$  and  $\tan \theta = -1.4$ . Round your answers to the nearest hundredth.
13. A point has coordinates  $(-6, -3)$ . Find its angle of reference. Round your answer to the nearest hundredth.
14. A point is at distance 6 from the origin and its angle of reference is  $240^\circ$ . Find the coordinates of the point  $P$ .
15. A point  $P$  different than  $(0, 0)$  lies in the line with equation  $y = 5x$ . What are the possible angles of reference for  $P$ ?
16. The segment with endpoints  $(-1, -2)$  and  $(3, 8)$  is a diameter of a circle.
  - (a) Give the equation of the circle in expanded form.
  - (b) What's the length of the circumference of this circle?
17. Find the common points of the circle  $x^2 - 6x + y^2 + 4y = 87$  and the line  $y = 3 - x$ .
18. Consider the circle  $C$  with equation  $x^2 + 2x + y^2 - 6y = 0$ .
  - (a) Verify that the point  $P(2, 4)$  lies on the circle  $C$ .
  - (b) Find an equation for the line tangent to  $C$  at  $P$ .
19. Find the standard form of the equation of the ellipse with foci at  $(0, -3)$  and  $(0, 3)$ , given that the sum of the distances of a point in the ellipse from the two foci is 10.
20. Sketch the graph of the ellipse:

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

The graph should correctly reflect the minor and major axis, the center and the foci.