

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 30
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Midterm
March 25, 2010

Name: KEY.

Directions: Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. This exam has a total of 100 points.

1. Find the domain for each of the following functions:

(a) (10 points) $f(x) = \frac{2x-1}{x^2-x-12}$ We need $x^2-x-12 \neq 0$

Now $x^2-x-12 = (x-4)(x+3)$.

So we need $x \neq 4$ and $x \neq -3$.

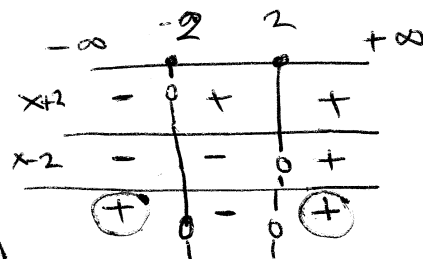


Thus the domain is $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$.

(b) (10 points) $g(x) = \sqrt{x^2-4}$ We need $x^2-4 \geq 0$. Now

$x^2-4 = (x-2)(x+2)$. We have the following table of signs:

So domain is $(-\infty, -2] \cup [2, \infty)$

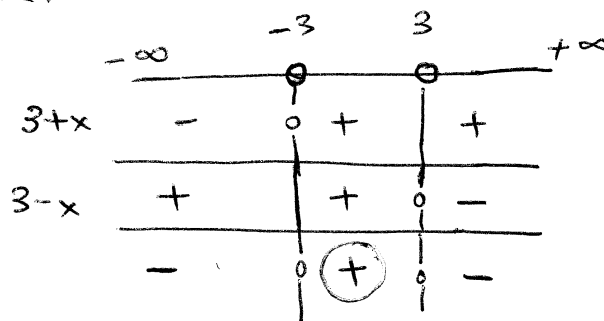


(c) (10 points) $h(x) = \log_2(9-x^2)$

We need $9-x^2 > 0$. Now $9-x^2 = (3+x)(3-x)$. So

we have the following table:

So domain is $(-3, 3)$.



2. (10 points) Find $f \circ g$, where $f(x) = \frac{2x-3}{5x+2}$ and $g(x) = \frac{x-1}{x+2}$

First the formula:

$$(f \circ g)(x) = f(g(x)) = \frac{2\left(\frac{x-1}{x+2}\right) - 3}{5\left(\frac{x-1}{x+2}\right) + 2} \cdot \frac{x+2}{x+2}$$

$$= \frac{2(x-1) - 3(x+2)}{5(x-1) + 2(x+2)}$$

$$= \frac{2x - 2 - 3x - 6}{5x - 5 + 2x + 4}$$

$$= \frac{-x - 8}{7x - 1} = -\frac{x+8}{7x-1}$$

Now the domain.

x is in the domain of $f \circ g$, when ^{both of} the following two conditions hold:

1) x is in domain of $g \iff x+2 \neq 0 \implies x \neq -2$

2) $g(x)$ is in domain of $f \iff 7x-1 \neq 0 \implies x \neq \frac{1}{7}$.

Combining these two conditions we have:



domain is

$$(-\infty, -2) \cup (-2, \frac{1}{7}) \cup (\frac{1}{7}, \infty).$$

3. (10 points) Let $f(x) = x^2 - 4x - 2$ with domain $(-\infty, 2]$, and $g(x) = 2 - \sqrt{x+6}$. Prove that f and g are a pair of inverse functions.

We will show that a) for x in domain of g , $f(g(x)) = x$
 b) for x in domain of f , $g(f(x)) = x$.

$$\begin{aligned} \text{a) } f(g(x)) &= (2 - \sqrt{x+6})^2 - 4(2 - \sqrt{x+6}) - 2 \\ &= 4 - 4\sqrt{x+6} + x + 6 - 8 + 4\sqrt{x+6} - 2 \\ &= x. \end{aligned}$$

$$\begin{aligned} \text{b) } g(f(x)) &= 2 - \sqrt{(x^2 - 4x - 2) + 6} \\ &= 2 - \sqrt{x^2 - 4x + 4} \\ &= 2 - \sqrt{(x-2)^2} \\ &= 2 - |x-2| \end{aligned}$$

Now if x is in the domain of f , $x-2 \leq 0$. So $|x-2| = -x+2$.
 So $g(f(x)) = 2 - (-x+2) = x$

4. (10 points) Find the formula, the domain and the range of f^{-1} , where

f as a relation is $y = \frac{-x+3}{4x-7}$. So f^{-1} is $x = \frac{-y+3}{4y-7}$

We solve for y : $x = \frac{-y+3}{4y-7} \Leftrightarrow x(4y-7) = -y+3$

$$\Leftrightarrow 4xy - 7x = -y + 3$$

$$\Leftrightarrow 4xy + y = 7x + 3$$

$$\Leftrightarrow (4x+1)y = 7x+3$$

$$\Leftrightarrow y = \frac{7x+3}{4x+1}$$

Thus the formula is

$$f^{-1}(x) = \frac{7x+3}{4x+1}$$

Domain of f^{-1} : we need $4x+1 \neq 0$
 So $x \neq -\frac{1}{4}$. Thus domain is $(-\infty, -\frac{1}{4}) \cup (-\frac{1}{4}, \infty)$

Range of f^{-1} is the domain of f .

$$4x-7 \neq 0 \Leftrightarrow x \neq \frac{7}{4}$$

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So ~~domain~~ range is $(-\infty, \frac{7}{4}) \cup (\frac{7}{4}, \infty)$.

5. (5 points) List all possible rational roots of the following polynomial, according to the "Rational Root Theorem".

$$p(x) = 6x^5 - 3x^4 + 7x^3 - 2x^2 + 8x - 12$$

Possible numerators: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Possible denominators: $\pm 1, \pm 2, \pm 3, \pm 6$

So possible solutions (after eliminating repetitions):
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$

6. (15 points) Solve the following equation:

$$x^5 - 5x^4 - x^3 + 11x^2 - 6 = 0$$

Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$.

We try them using synthetic division:

$$\begin{array}{r|rrrrrr} 1 & 1 & -5 & -1 & 11 & 0 & -6 \\ & & 1 & -4 & -5 & 6 & 6 \\ \hline 1 & 1 & -4 & -5 & 6 & 6 & \boxed{0} \checkmark \\ & & 1 & -3 & -8 & -2 & \\ \hline 1 & 1 & -3 & -8 & -2 & \boxed{4} & X \end{array}$$

$$\begin{array}{r|rrrrrr} -1 & 1 & -4 & -5 & 6 & 6 \\ & & -1 & 5 & 0 & -6 \\ \hline -1 & 1 & -5 & 0 & 6 & \boxed{0} \checkmark \\ & & -1 & 6 & -6 & \\ \hline 1 & 1 & -6 & 6 & \boxed{0} \checkmark \end{array}$$

So the L.H.S. factors as:

$$(x-1)(x+1)^2(x^2-6x+6)$$

So we have

$$\boxed{x=1} \text{ or } \boxed{x=-1} \text{ (double sol.)}$$

$$\text{or } x^2 - 6x + 6 = 0$$

$$D = 36 - 24 = 12.$$

$$\begin{aligned} \text{Solutions: } x &= \frac{6 \pm \sqrt{12}}{2} \\ &= \frac{6 \pm 2\sqrt{3}}{2} \\ &= \boxed{3 \pm \sqrt{3}} \end{aligned}$$

In sum, $x=1$ or $x=-1$,
 or $x=3+\sqrt{3}$
 or $x=3-\sqrt{3}$

7. (20 points) Solve the following inequality:

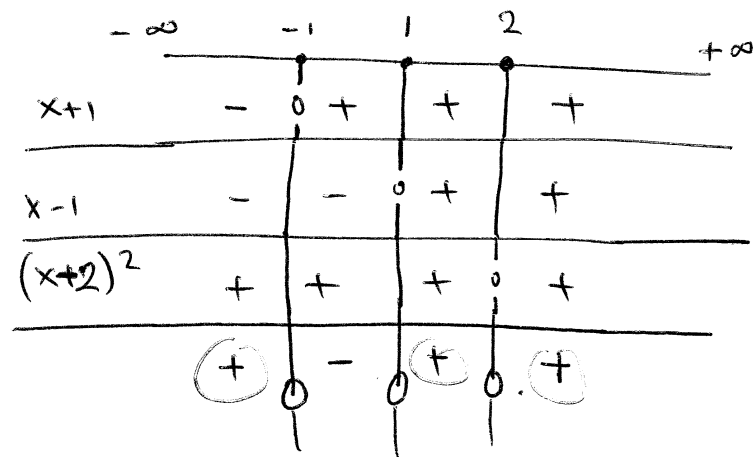
$$x^4 - 4x^3 + 3x^2 + 4x - 4 \geq 0$$

First we factor. We check possible rational roots: $\pm 1, \pm 2$ and ± 4

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 3 & 4 & -4 \\ & & 1 & -3 & 0 & 4 \\ \hline 1 & 1 & -3 & 0 & 4 & 0 \\ & & 1 & -2 & -2 & \\ \hline 1 & -2 & -2 & 0 & & \end{array}$$

$$\begin{aligned} \text{So } x^4 - 4x^3 + 3x^2 + 4x - 4 &= (x-1)(x+1)(x^2 - 4x + 4) \\ &= (x-1)(x+1)(x-2)^2 \end{aligned}$$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 4 \\ & & -1 & 4 & 4 \\ \hline 1 & -4 & 4 & 0 & \end{array}$$



So solution is $(-\infty, -1] \cup [1, \infty)$