BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 30 — Midterm (Take II)
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Name: KEY

Directions: Write your answers in the provided space. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. This exam has a total of 110 points. However the perfect score is 100 points, there are 10 points of extra credit.

1. Find the domain for each of the following functions:

(a) (10 points) $f(x) = \dfrac{x^2 - 3x + 1}{x^2 + 4x + 3}$

$x^2 + 4x + 3 \neq 0$

$(x+3)(x+1) \neq 0$

$x \neq -3, \quad x \neq -1$

Domain $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$

(b) (10 points) $g(x) = \sqrt{-x^2 + x + 6}$

$-x^2 + x + 6 \geq 0$

$x^2 - x - 6 \leq 0$

$(x-3)(x+2) \leq 0$

	$-\infty$	-2		3	$+\infty$
$x+2$	$-$	0	$+$		$+$
$x-3$	$-$		$-$	0	$+$
	$+$	0	$-$	0	$+$

Domain $[-2, 3]$

(c) (10 points) $h(x) = \log_{42}(x^2 + 4)$

$x^2 + 4 > 0$. All real numbers

Domain $\mathbb{R}$.

2. (15 points) Find $f \circ g$, where $f(x) = \dfrac{x^2+1}{x^2-4}$ and $g(x) = \sqrt{2-4x}$

Formula

$$(f \circ g)(x) = \frac{(\sqrt{2-4x})^2 + 1}{(\sqrt{2-4x})^2 - 4} = \frac{2-4x+1}{2-4x-4} = \boxed{\frac{-4x+3}{-4x-2}}$$

Domain: x in domain of g : $2-4x \geq 0 \Rightarrow 2 \geq 4x \Rightarrow \boxed{\frac{1}{2} \geq x}$

$g(x)$ in domain of f : $-4x-2 \neq 0 \Rightarrow x \neq -\frac{1}{2}$

So domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{2}]$.

3. (15 points) Prove that $f(x) = \dfrac{3x-2}{2x+1}$ and $g(x) = \dfrac{x+2}{3-2x}$ are a pair of inverse functions.

$$\boxed{\begin{aligned} f(g(x)) &= \frac{3\left(\frac{x+2}{3-2x}\right) - 2}{2\left(\frac{x+2}{3-2x}\right) + 1} \\ &= \frac{3(x+2) - 2(3-2x)}{2(x+2) + 3 - 2x} \\ &= \frac{3x+6-6+4x}{2x+4+3-2x} \\ &= \frac{7x}{7} \\ &= x \end{aligned}}$$

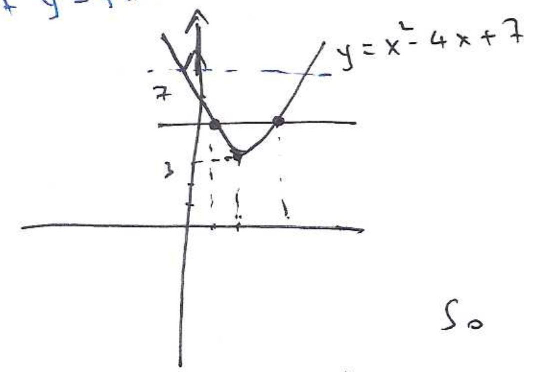
$$\begin{aligned} g(f(x)) &= \frac{\frac{3x-2}{2x+1} + 2}{3 - 2\,\frac{3x-2}{2x+1}} \\ &= \frac{3x-2+4x+2}{6x+3-6x+4} \\ &= \frac{7x}{7} \\ &= x. \end{aligned}$$

4. Consider the function $f(x) = x^2 - 4x + 7$.

(a) (5 points) Show that f is not one-to-one.

Complete the square: $y = x^2 - 4x + 7 \iff y = (x-2)^2 + 3$

So the graph of $y = f(x)$



OR, for example

$f(0) = 7$

$f(4) = 16 - 16 + 7 = 7$

So two different x-values give the same y-value. $\Rightarrow$ f not one to one.

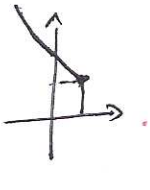
(b) (15 points) Restrict the domain of f in such a way that it becomes one-to-one. Then find the inverse of the restricted function.

$y = (x-2)^2 + 3$

Either $(-\infty, 2]$

or $[2, \infty)$.

or



Inverse $x = (y-2)^2 + 3$

$(y-2)^2 = x + 3$

$y - 2 = \pm\sqrt{x+3}$

$y = 2 \pm \sqrt{x+3}$

So restricting domain $(-\infty, 2]$

we have $f^{-1}(x) = 2 - \sqrt{x+3}$

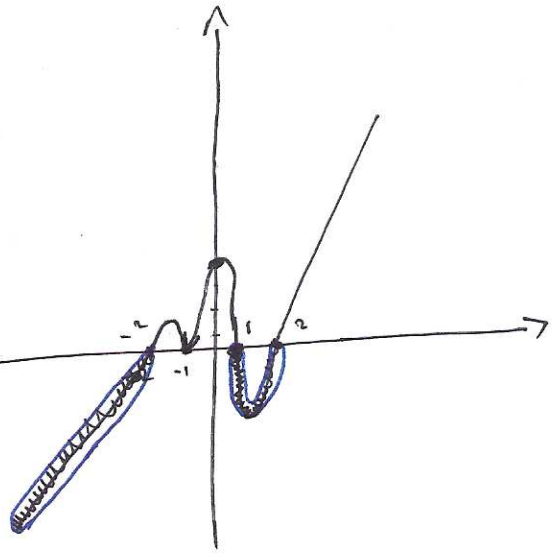
Restricting domain to $[2, \infty)$:

$f^{-1}(x) = 2 + \sqrt{x+3}$.

5. (a) (15 points) Sketch a rough graph of the following function. The graph should correctly reflect the end behavior, the behavior near zeros and the number of urning points. The yintercept should also be correctly marked.

$$p(x) = \boxed{x^5} + x^4 - 5x^3 + 4x - 5x^2 + 4$$

1	1	1	-5	-5	4	4
		1	2	-3	-8	-4
1	1	2	-3	-8	-4	$\boxed{0}$
		1	3	0	-8	
	1	3	0	-8	-12	X
-1	1	2	-3	-8	-4	
		-1	-1	4	4	
-1	1	1	-4	-4	$\boxed{0}$	
		-1	0	4		
	1	0	-4	$\boxed{0}$		



$x^2 - 4 = (x-2)(x+2)$

So $p(x) = (x+1)^2 (x+1)(x+2)(x-2)$

$x=1$ $\quad x=-1$ $\quad x=-2$ $\quad x=2$

(b) (5 points) Solve the inequality: $x^5 + x^4 - 5x^3 + 4x - 5x^2 + 4 < 0$

From the graph we see that $p(x)$ is negative at $(-\infty, -2) \cup (1, 2)$.

OR table of signs &

	$-\infty$	-2		-1		1		2	$+\infty$
$x+2$	-	0	+		+		+		+
$(x+1)^2$	+		+	0	+		+		+
$x-1$	-		-		-	0	+		+
$x-2$	-		-		-		-	0	+
$p(x)$	-	0	+	0	+	0	-	0	+

$(-\infty, -2] \cup (1,2)$.

6. (10 points) For a polynomial $p(x)$ the graph of $y = p(x)$ has the following properties:

- The only x–intercepts are at $x = -1$, $x = 2$ and $x = 3$.
- The y–intercept is at $y = -12$
- As $x \to \infty$, $p(x) \to \infty$ and as $x \to -\infty$, $p(x) \to \infty$.

Give an example of such a polynomial.

$$(x+1)(x-2)^2(x-3) = (x+1)(x^2-4x+4)(x-3)$$
$$= (x^2-2x-3)(x^2-4x+4)$$
$$= x^4-4x^3+4x^2-2x^3+8x^2-8x-3x^2+12x-12$$
$$= \boxed{x^4-6x^3-x^2+4x-12}$$

Explanation: $x-1$, $x-2$, and $x-3$ are factors.
By the end behaviour I know that the degree has to be even. So one of the roots should be double. If we take $x=2$ to be double, all conditions are satisfied.