Additional Review Questions for the Math 30 final

Nikos Apostolakis

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1. Find the domain of each of the following functions:
   (a) $f(x) = \ln(x^2 + x - 6)$ $(-\infty, -3) \cup (2, \infty)$
   (b) $g(x) = \log_3 \dfrac{x+3}{x-4}$ $(-\infty, -3) \cup (4, \infty)$
   (c) $h(x) = \sqrt{x^2 - 8x + 16}$ $(-\infty, 4) \cup (4, \infty)$
   (d) $h(x) = \sqrt{-x^3 - 2x^2 + 9x + 18}$ $(-\infty, -3] \cup [-2, 3]$
   (e) $k(x) = \dfrac{2x-3}{2x^3 - x^2 - 7x + 6}$ $(-\infty, -2) \cup (-2, 1) \cup (1, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

2. For each of the following pair of functions find the formula and the domain for $f \circ g$ and $g \circ f$.
   (a) $f(x) = \dfrac{2x-3}{x-2}$, $g(x) = \dfrac{2x}{3x-1}$

   Answer. Domain of $f \circ g$ is $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$. The formula for $f \circ g$ is
   $(f \circ g)(x) = \dfrac{5x-3}{4x-2}$
   Domain of $g \circ f$ is $(-\infty, \frac{7}{5}) \cup (\frac{7}{5}, 2) \cup (2, \infty)$. The formula for $g \circ f$ is
   $(g \circ f)(x) = \dfrac{4x-6}{5x-7}$ □

   (b) $f(x) = \dfrac{3}{x^2-4}$, $g(x) = \sqrt{x+2}$

   Answer. Domain of $f \circ g$ is $[-2, 2) \cup (2, \infty)$. The formula for $f \circ g$ is
   $(f \circ g)(x) = \dfrac{3}{x-2}$
   Domain of $g \circ f$ is $(-\infty, 2) \cup [-\frac{\sqrt{10}}{2}, \frac{\sqrt{10}}{2}] \cup (2, \infty)$. The formula for $g \circ f$ is
   $(g \circ f)(x) = \sqrt{\dfrac{2x^2-5}{x^2-4}}$ □

   (c) $f(x) = x^2 - 2x + 4$ and $g(x) = 1 - \sqrt{x-3}$

   Answer. Domain $f \circ g$ is $[3, \infty)$. The formula for $f \circ g$ is $(f \circ g)(x) = x$.
   Domain of $g \circ f$ is $\mathbb{R}$. The formula for $g \circ f$ is $(g \circ f)(x) = 1 - |x - 1|$. □

3. For each of the following functions find the domain, the range and the inverse function.

(a) $g(x) = \sqrt{3x-4}$

Answer. Domain is $[\frac{4}{3}, \infty)$. Range is $[0, \infty)$. The inverse function is $g^{-1}(x) = \dfrac{x^2+4}{3}$ □

(b) $f(x) = \dfrac{2x}{3x-1}$

Answer. Domain is $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$. Range is $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$. The inverse function is $f^{-1}(x) = \dfrac{x}{3x-2}$. □

(c) $k(x) = 2x^2 - 4x + 9$, with domain $(-\infty, 1]$

Answer. Domain is $(-\infty, 1]$. Range is $[7, \infty)$. The inverse is $k^{-1}(x) = \dfrac{2-\sqrt{2x-14}}{2}$ □

(d) $f(x) = -x^2 + 6x - 8$, with domain $[3, \infty)$

Answer. Domain is $[3, \infty)$. Range is $(-\infty, 1]$. Inverse is $f^{-1}(x) = 3 + \sqrt{1-x}$ □

(e) $h(x) = 2^{4x-5}$

Proof. Domain is $\mathbb{R}$. Range is $(0, \infty)$. Inverse is $h^{-1}(x) = \dfrac{\log_2 x + 5}{4}$ □

(f) $g(x) = \ln(5x-2) + 3$

Answer. Domain is $(\frac{2}{5}, \infty)$. Range is $\mathbb{R}$. Inverse is $g^{-1}(x) = \dfrac{e^{x-3}+2}{5}$. □

4. Solve:

   (a) $x^4 - x^3 - 7x^2 + x + 6 = 0$ $\quad x = -2, \quad x = -1, \quad x = 1, \quad x = 3$

   (b) $x^4 - 3x^3 + 3x^2 + 12x - 28 = 0$ $\quad x = -2, \quad x = 2, \quad x = \dfrac{3+i\sqrt{19}}{2}, \quad x = \dfrac{3-i\sqrt{19}}{2}$

   (c) $x^3 - 6x^2 + 11x - 6 \geq 0$ $\quad [1, 2] \cup [3, \infty)$

5. Solve each of the following equations:

   (a) $e^{2x} - 3e^x + 2 = 0$ $\quad x = \ln 1, \quad x = \ln 2$

   (b) $2^{4x} - 10 \cdot 2^{2x} + 9 = 0$ $\quad x = 0, \quad x = \log_2 3$

   (c) $\log_3(x-1) + \log_3(x+2) = 1$ $\quad x = \dfrac{\sqrt{21}-1}{2}$

6. Solve the following equations. You should give *all* solutions.

   (a) $\cos^2 x - \cos x = 0$

   Answer. Three families of solutions: $x = 2k\pi$, $x = \frac{\pi}{2} + 2k\pi$, $x = -\frac{\pi}{2} + 2k\pi$, where in each formula k is an arbitrary integer. □

   (b) $2\sin^2 x - \sin x - 1 = 0$

Answer. Three families of solutions: $x = \frac{\pi}{2} + 2k\pi$, $x = -\frac{\pi}{6} + 2k\pi$, $x = \frac{7\pi}{6} + 2k\pi$, where in each formula k is an arbitrary integer. □

(c) $\cos 3x = \frac{\sqrt{3}}{2}$

Answer. Two families of solution $x = \frac{12k\pi \pm \pi}{18}$, where k is an arbitrary integer. □

(d) $4\sin^4 x + 4\sin^3 x - \sin^2 x - \sin x = 0$

Answer. $x = 2k\pi$, $x = 2k\pi + \pi$, $x = \frac{\pi}{2} + 2k\pi$, $x = \frac{\pi}{4} + 2k\pi$, $x = -\frac{\pi}{4} + 2k\pi$, $x = \frac{5\pi}{4} + 2k\pi$, $x = \frac{3\pi}{4} + 2k\pi$, where in each formula k stands for an arbitrary integer. □

7. For each of the sinusoidal curves in Figures 1 and 2 find an equation of the form:
   (a) $A\sin(Bx + C)$ with $A > 0$
   (b) $A\sin(Bx + C)$ with $A < 0$
   (c) $A\cos(Bx + C)$ with $A > 0$
   (d) $A\cos(Bx + C)$ with $A < 0$

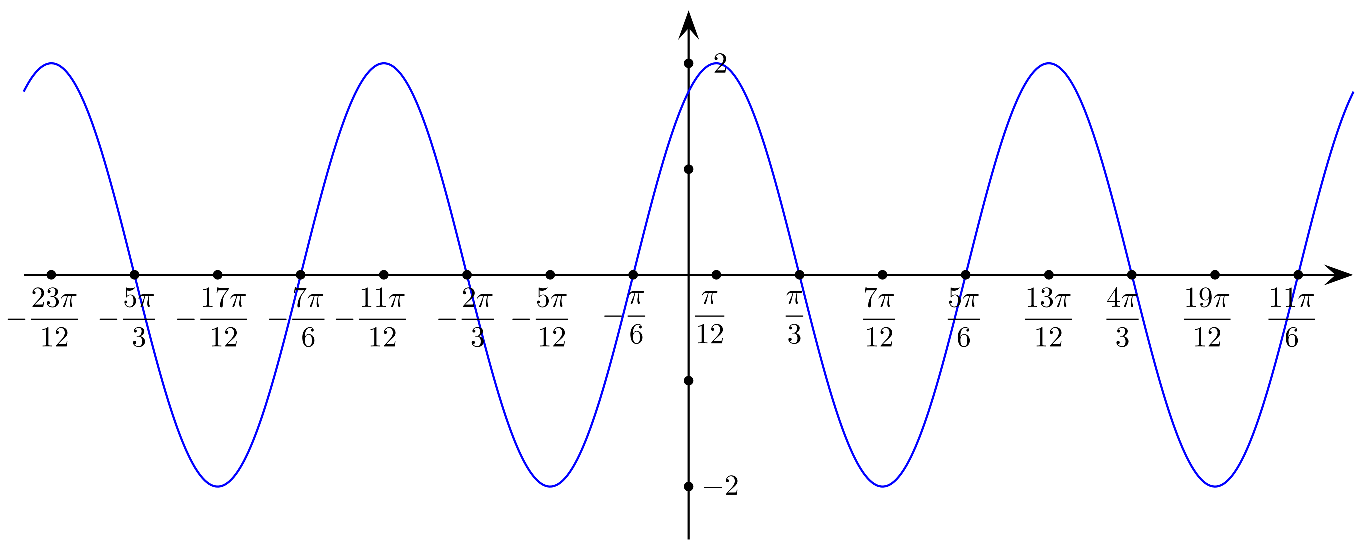

Figure 1: A sinusoidal curve

Answer. The curve in Figure 1 has equations:

1. $y = 2\sin(2x + \frac{\pi}{3})$
2. $y = -2\sin(2x - \frac{2\pi}{3})$
3. $y = 2\cos(2x - \frac{\pi}{6})$
4. $y = -2\cos(2x + \frac{5\pi}{6})$

The curve in Figure 2 has equations:

1. $y = 3\sin(2x - \frac{\pi}{6})$
2. $y = -3\sin(2x + \frac{5\pi}{6})$
3. $y = 2\cos(2x - \frac{2\pi}{3})$
4. $y = -2\cos(2x + \frac{\pi}{3})$

□

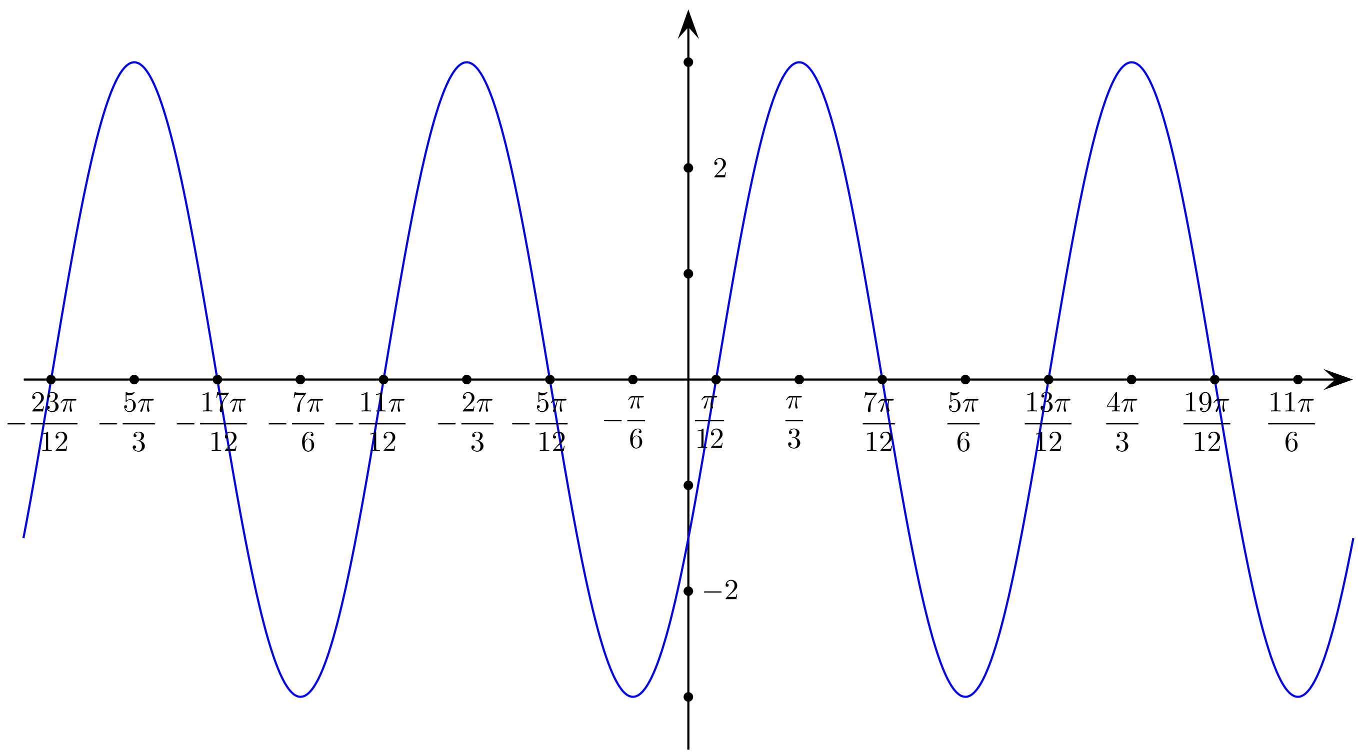

Figure 2: Another sinusoidal curve