Fifth Quiz for CSI35
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Directions: This quiz is due Thursday April 23, at 6:00 PM. Please make sure to *justify* all your answers. **No credit will be given for unjustified answers.**

1. How many equivalence relations are there on the sete $\{1, 2, 3, 4\}$? Provide an explicit list.

2. How many equivalence relations are there on a set with five elements?

3. Consider the standard linear order on $\mathbb{R}$. Which of the following sets has minimum, maximum, lower bound, upper bound?

   (a) $(0, \infty)$

   (b) $[-3, 5)$

   (c) $(-\infty, 6)$

   (d) $\{x \in \mathbb{R} : x^2 < 2\}$

   (e) $\{x \in \mathbb{R} : x^3 > 2\}$

4. Consider the poset $(\mathcal{P}(A), \subseteq)$ where $A = 0, 1, 2, 3$. Is there a minimum element? Is there a maximum element?

5. Consider the poset $(\mathcal{S}, \subseteq)$ where $\mathcal{S}$ is the set of non empty proper subsets of $\{0, 1, 2, 3\}$. Does $\mathcal{S}$ have minimum or maximum elements? How about minimal or maximal elements?

6. Let $(P, \preceq)$ be a poset that has only one minimal element m. Is m necessarily minimum? If your answer is affirmative then you should prove it, if it is negative then you should provide a counterexample.

7. Let $A = \mathbb{N} \setminus \{0, 1\}$ and consider the poset $(A, |)$, where $|$ is the divisibility relation. If $B = \{6, 15\}$

   (a) find all lower bounds of B. Is there a largest lower bound?

   (b) find all lower bounds of B. Is there a least upper bound?

8. **Extra Credit:** On a 3×4 chessboard there are three white knights on the top row and three black knights on the bottom row, as shown in the following picture. Using only legal moves, interchange the

W	W	W
B	B	B

black and white knights in as few moves as you can.