Fourth Quiz for CSI35

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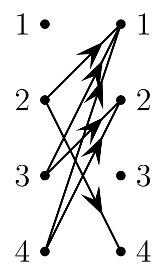
Directions: This quiz is due Thursday March 26, at 6:00 PM.

1. Consider the relation R whose bipartite digraph is shown below



   (a) Find the digraph of R.

   (b) Write R as a set of ordered pairs.

   (c) Determine whether R is reflexive, symmetric antisymmetric, or transitve.

   (d) Find $R \circ R$.

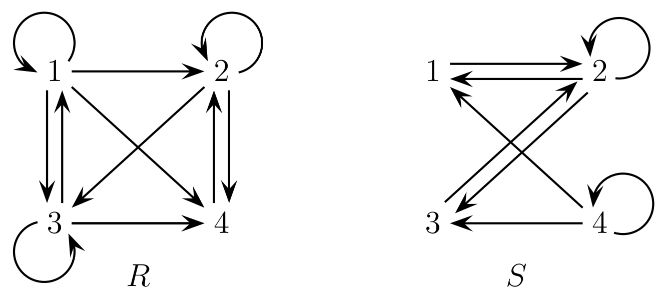
2. Let R be the "auntle" relation on the set of all humans: $(a, b) \in R$ if and only if, a is an aunt or an uncle of b (in other words R is the composition $S \circ P$ where P is the "parent relation" and S is the "sibling" relation). What are the compositions $P \circ R$ and $R \circ P$?

3. Let $A = \{0, 1\}$.

   (a) How many relations are there on A? List all of them.

   (b) Which of the relations you listed in par (a) are reflexive? Which are symmetric? Which are antisymmetric? Which are transitive?

4. Consider the relations R, and S on the set $\{1, 2, 3, 4\}$ represented by the digraphs:



(a) Find the matrices M_S and M_R of M and S respectively.

(b) Use these matrices to compute the compositions $R \circ S$ and $S \circ R$.

(c) Draw the digraphs that represent $R \circ S$ and $S \circ R$.

5. Let R be a relation on A. Is it possible R to be a function and reflexive? If yes give an example, if no explain why not.

6. Which of the following relations defined on the set of all people are equivalence relations. Justify your answers:

   (a) $(a, b) \in R$ iff a has the same parents as b.

   (b) $(a, b) \in R$ iff a is parent of b.

   (c) $(a, b) \in R$ iff a lives in the same town as b.

   (d) $(a, b) \in R$ iff a lives one floor above b.

   (e) $(a, b) \in R$ iff a is an acquaintance of b.

7. Consider the following relation on $\mathbb{R}$, the set of reall numbers

$$(a, b) \in R \Longleftrightarrow |a| = |b|$$

   Prove that R is an equivalence relation.

8. Consider the relation R defined on the set of all positive real numbers as follows:

$$(a, b) \in R \quad \text{iff} \quad \frac{a}{b} \in \mathbb{Q}\,,$$

   where $\mathbb{Q}$ stands for the set of rational numbers. Prove that R is an equivalence relation.

9. Let Δ_n be the set of all diagonal $n \times n$ matrices with real elements, i.e. a matrix $A = (a_{ij})$ is in Δ_n iff and only if, $\forall i, j \quad i \neq j \Longrightarrow a_{ij} = 0$. Consider the relation R defined on the set M_n of all $n \times n$ matrices by

$$(A, B) \in R \Longleftrightarrow A - B \in \Delta_n$$

   (a) Prove that R is an equivalence relation.

   (b) What is the equivalence class of the identity matrix I_n?

10. Consider the relation defined on the set of ordered pairs of natural numbers (i.e. on the set $\mathbb{N} \times \mathbb{N}$) as follows:

$$((m,n),(k,l)) \in R \Longleftrightarrow m+l=k+n$$

   (a) Prove that R is an equivalence relation.

   (b) Find the equivalence class of $(5,6)$.

11. How many equivalence relations are there on the set $\{1,2,3,4,5\}$?