Third Quiz for CSI35

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Directions: This quiz is due Thursday March 19, at 6:00 PM.

1. How many $n \times n$ zero-one matrices are both symmetric and antisymmetric?
2. How many $n \times n$ zero-one matrices are both symmetric and reflexive?
3. How many $n \times n$ zero-one matrices are both antisymmetric and reflexive?
4. The *complementary bipartite digraph* of a digraph G is a new bipartite digraph $\bar{G}$ defined as follows:
   - The departure points and destinations of $\bar{G}$ are the same as the departure points and destinations of G respectively.
   - For a departure point x and a destination y there is an edge in $\bar{G}$ from x to y exactly when there is no edge in G from x to y.

   (a) If $A = (a_{ij})$ is the matrix of a bipartite digraph G, describe $\bar{A}$, the matrix of $\bar{G}$.

   (b) What can you say about $\bar{G}$ if G is
      i. Symmetric.
      ii. Antisymmetric.
5. An $n \times n$ zero-one matrix is called *asymmetric* if

$$\forall i, j \in \{1, \ldots, n\} \quad a_{ij} = 1 \Longrightarrow a_{ji} = 0$$

   (a) Express this condition in terms of the corresponding digraph. In other words, when is the matrix of digraph asymmetric?

   (b) Show that an asymmetric matrix is antisymmetric.

   (c) Is it true that every antisymmetric matrix is asymmetric?

   (d) How many asymmetric matrices are there?
6. Which of the following sentences are true?

(a) The empty digraph is symmetric.

(b) The empty digraph is reflexive.

(c) The empty digraph is antisymmetric.

(d) The empty digraph is asymmetric.

7. **Extra Credit:** Let D_1 and D_2 be two symmetric digraphs. Prove that $D_1 \circ D_2$ is also symmetric.