

Fourth Quiz for Math 30, section 6432

Directions: You should fully justify your answers. Do all your work on separate paper, and make sure to *print* your name in the first sheet and staple all the sheets together. **Unstapled, loose pieces of paper will not be graded.** This quiz is due Monday March 10, at 6:00 PM.

1. Find the remainder of the division:

$$\frac{x^{100} - x^2 + 7}{x - 1}$$

2. Find a fourth degree *monic* polynomial that has roots at $x = -3$, $x = -2$, $x = 1$, $x = 2$.
3. Can you find a third degree polynomial with real coefficients with roots at $x = 2 + i$, $x = 2$, and $x = -1$?
4. Find all roots of each of the following polynomials:
- (a) $x^4 + 2x^3 - 7x^2 - 8x + 12$
 - (b) $x^4 - x^3 - x^2 - x - 2$
 - (c) $2x^3 + 7x^2 + 6x - 5$

5. List all possible rational roots of each of the following polynomials:

- (a) $x^3 + 3x^2 - 5x - 60$
- (b) $2x^7 - 5x^6 + 2x^2 + 3x - 21$
- (c) $12x^4 - 15x^3 - 4x^2 + x + 6$

6. Given that $x^2 + 3$ is a factor of

$$p(x) = x^4 - 4x^3 - 12x - 9$$

factor $p(x)$ completely.

7. **Extra Credit:** Prove that the following polynomial has at least one irrational root.

$$x^3 - x^2 - x + 3$$