Second Quiz for Math 30, section 6432

The answers

1. Let $f(x) = \sqrt{x+1}$ and $g(x) = x + 3$. Find the domain and the formula for

   (a) $f + g$

   Answer. Domain is $[-1, \infty)$. The formula is $(f+g)(x) = \sqrt{x+1} + x + 3$. □

   (b) $f \cdot g$

   Answer. Domain is $[-1, \infty)$. The formula is $(f \cdot g)(x) = (x+3)\sqrt{x+1}$ □

   (c) $\dfrac{f}{g}$

   Answer. Domain is $[-1, \infty)$. The formula is $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x+1}}{x+3}$. □

   (d) $f \circ g$

   Answer. Domain is $[-4, \infty)$. The formula is $(f \circ g)(x) = \sqrt{x+4}$. □

2. Let $f(x) = \dfrac{x+2}{x}$ and $g(x) = \dfrac{2}{x-1}$.

   (a) Find $f \circ g$ and $g \circ f$.

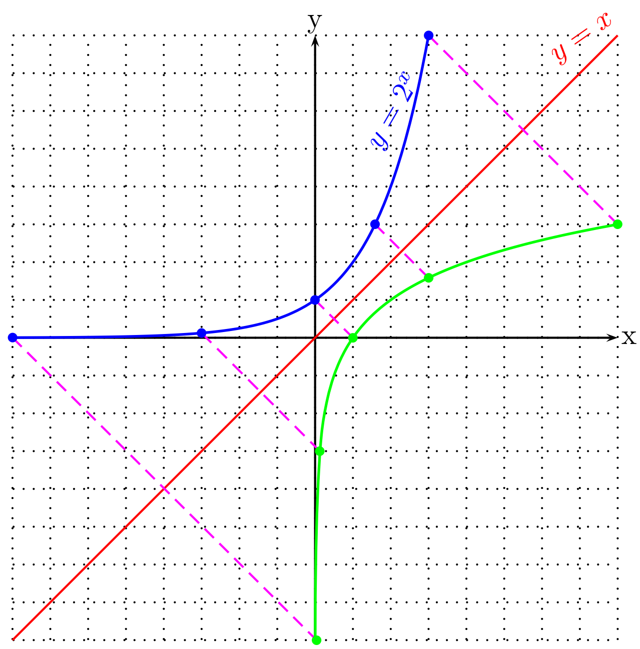
   Answer. $(f \cdot g)(x) = x$ and $(g \cdot f)(x) = x$. □

   (b) What does your result mean?

   Answer. f and g is a pair of inverse functions. □

   (c) What is the range of f?

   Answer. The range of f equals the domain of its inverse g. So the range of g is $(-\infty, 1) \cup (1, \infty)$. □



3. The graph of the function $f(x) = 2^x$ is shown bellow together with the graph of the diagonal $y = x$. Explain why f has an inverse function and then sketch the graph of $y = f^{-1}(x)$ on the same grid.

   Answer. f passes the "horizontal line test" and therefore is invertible. To find the graph of its inverse function f^{-1} we relect the graph of f with respect to the diagonal. □

4. For each of the following functions find the domain, the range and the inverse function.

   (a) $f(x) = 4x - 5$

   Answer. Domain is $(-\infty, \infty)$, range is $(-\infty, \infty)$ and the inverse function is given by the formula $f^{-1}(x) = \dfrac{x+5}{4}$. □

   (b) $g(x) = \dfrac{5}{x-1}$

   Answer. Domain is $(-\infty, 1) \cup (1, \infty)$, range is $(-\infty, 0) \cup (0, \infty)$ and the inverse function is given by the formula $g^{-1}(x) = \dfrac{x+5}{x}$. □

   (c) $h(x) = x^3 - 4$

   Answer. Domain is $(-\infty, 1) \cup (1, \infty)$, range is $(-\infty, 0) \cup (0, \infty)$ and the inverse function is given by the formula $h^{-1}(x) = \sqrt[3]{x+4}$. □

   (d) $k(x) = \sqrt{-x}$

   Proof. Domain is $(-\infty, 0]$, range is $[0, \infty)$ and the inverse function is given by the formula $k^{-1}(x) = -x^2$. □

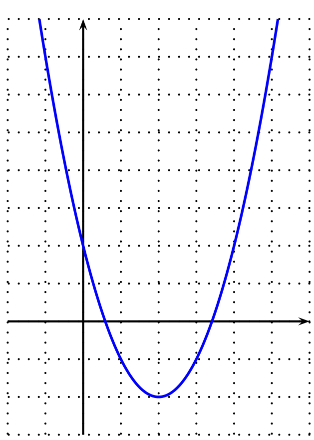
5. **Extra Credit:** Consider the following function:

$$f(x) = x^2 - 4x + 2$$

   (a) Use the method of *completing the square* to put this quadratic function in standard form.

   Answer. The standard form of the equation is $y = (x-2)^2 - 2$ □

   (b) Graph $y = f(x)$.

   Answer.  □

(c) Prove that this function does not have an inverse function.

Answer. The graph of f does not pass the horizontal line test. Therefore f is not invertible. □

(d) How can we restrict the domain of f so that it has an inverse function?

Answer. If we restrict the domain of f to be $[2, \infty)$ the graph of f will pass the horizontal line test. An other choise of restrited domain is $(-\infty, 2]$. □

(e) After the domain of f has been restricted as in part (d) find f^{-1}.

Answer. If we restrict the domain to $[2, \infty)$ then the formula for the inverse is $f^{-1}(x) = \sqrt{x+2} - 2$. If we restrict the domain to $(-\infty, 2]$ the formula for the inverse is $f^{-1}(x) = 2 - \sqrt{x+2}$. □