First Quiz for Math 30, section 6432

The answers

1. Consider the following relation:

$$\{(1,2),(3,1),(2,3),(5,3),(4,1),(2,4)\}$$

   (a) Find its domain and its range.

   Answer. Domain is $\{1,2,3,4,5\}$ and range is $\{1,2,3,4\}$. □

   (b) Is this relation a function? Justify your answer.

   Answer. This relation is not a function because the pairs $(2,3)$ and $(2,4)$ which are in the relation have the same first coordinate but different second coordinates. This cannot happen in a relation that is a function. □

2. Find the difference quotient for the function $f(x) = 3x^2 - 4x + 5$.

   Answer.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)^2-4(x+h)+5-(3x^2-4x+5)}{h}\\ &= \frac{3x^2+6hx+3h^2-4x-4h+5-3x^2+4x-5}{h}\\ &= \frac{6hx+3h^2-4h}{h}\\ &= 6x+3h-4\end{aligned}$$

   □

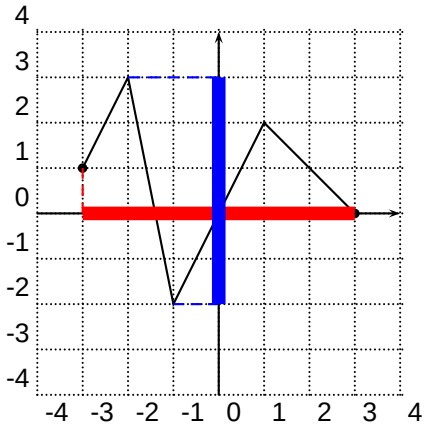
3. Find the largest possible domain for the function $f(x) = \dfrac{2x}{x^2+8x+15}$.

   Answer. The formula of the function involves denominators. So we have to make sure that the denominator is not zero. In other words we have to make sure that $x^2+8x+15 \neq 0$. So we must have $x \neq -3$ and $x \neq -5$. So the domain is

$$(-\infty,-5)\cup(-5,-3)\cup(-3,\infty)$$

   □

4. Consider the function h whose graph is shown. Find:



(a) The domain and the range.

Answer. Domain is $[-3, 3]$. Range is $[-2, 3]$. □

(b) Intervals on which h is increasing, decreasing, or constant.

Answer. h is increasing on $(-3, -2)$ and $(-1, 1)$ and decreasing on $(-2, -1)$ and $(1, 3)$. □

(c) Relative minima and maxima.

Answer. There are two relative maxima: at $x = -2$ with value 3 and at $x = 1$ with value 2. There are three relative minima: at $x = -3$ with value 1, at $x = -1$ with value -2 and at $x = 3$ with value 0. □

5. The graph of the function g is obtained by shifting the graph of the function $f(x) = 2x^3$ three units to the right along the x–axis and four units downwards along the y–axis. Find a formula for $g(x)$. (**You don't need to graph** g).

Answer. $g(x) = 2(x-3)^3 - 4$ (Or after expanding $g(x) = 2x^3 - 18x^2 + 54x - 58$). □

6. Use the graph of the function $f(x) = x^3$ to graph

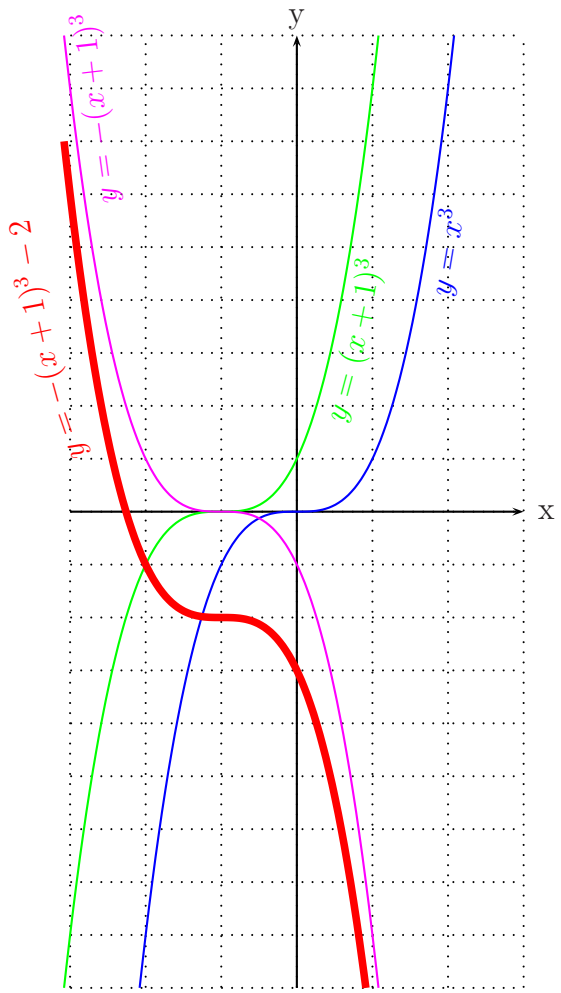
$$y = -(x+1)^3 - 2$$

.

Answer. To get the graph of $y = -(x+1)^3 - 2$ we can follow these steps:

- Shift the graph of $y = x^3$ by -1 unit along the x–axis to get the graph of $y = (x+1)^3$.
- Then reflect the graph of $y = (x+1)^3$ with respect to the x–axis to get the graph of $y = -(x+1)^3$, and finally
- shift the graph of of $y = -(x+1)^3$ by -2 units along the y–axis to get the graph of $y = -(x+1)^3 - 2$.

The final graph along with all the intermediate steps is shown in the following figure:


y
x
$y = -(x+1)^3 - 2$
$y = -(x+1)^3$
$y = (x+1)^3$
$y = x^3$

□