The answers to the second exam

1. Two fair dice are rolled. Find the probability that we get a sum of at least 9.

   Answer. All possible outcomes are listed below:

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

$$p(\Sigma \geq 9) = \frac{10}{36} = \frac{5}{18}$$

$\square$

2. Two cards are drawn without repetition from a standard 52–card deck. What is the probability that both cards are figures? (A figure is a King, a Queen, or a Jack.)

   Answer. There are 12 figures in a deck of cards. So the probability is:

$$\begin{aligned} p(2 \text{ figures}) &= p(\text{first is figure})p(\text{second is figure } \textit{given} \text{ first is figure}) \\ &= \frac{12 \cdot 11}{52 \cdot 51} \\ &= \frac{11}{221} \end{aligned}$$

$\square$

3. Consider the following discrete probability distribution:

x	1	2	4	6
$p(x)$	.3	.4	.1	.2

   Calculate its expected value and standard deviation.

   Answer.

x	$p(x)$	$xp(x)$	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 p(x)$
1	.3	0.3	-1.7	2.89	0.867
2	.4	0.8	-0.7	0.49	0.196
4	.1	0.4	1.3	1.69	0.169
6	.2	1.2	3.3	10.89	2.178
		2.7			3.41

   So, $\mu = 2.7$, $\sigma^2 = 3.41$ and $\sigma = \sqrt{3.41} = 1.85$

$\square$

4. A basketball player makes 80% of the free throws she shoots. If she shoots 10 free throws what is the probability that she will make exactly 8 shots?

Answer. Let X be the random variable that counts how many throws she makes. Then X follows a binomial probability distribution with $p = .80$ and $n = 10$ and we need to calculate $p(X = 8)$. We can use the formula:

$$\begin{aligned} p(X = 8) &= \binom{10}{8}(.80)^8(.20)^2 \\ &\approx 0.302 \end{aligned}$$

(Note that we could also get this value from the table of the binomial distribution.) □

5. Alice and Bob play the following game: they flip two coins, if both are heads then Alice wins, otherwise Bob wins. If they play 7 times what is the probability that Alice wins at most 3 times?

Answer. We first calculate the probability p that Alice will win in one run of the game. The events "heads in the first coin" and "heads in the second coin" are independent. Therefore the probability of getting two heads is:

$$\begin{aligned} p &= p(\text{first head } \textit{and} \text{ second head}) \\ &= p(\text{first head})p(\text{second head}) \\ &= \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{1}{4} \\ &= 0.25 \end{aligned}$$

If we call the outcome "Alice wins" success we have a binomial distribution with $p = .25$ and $n = 7$; and we need to compute the probability $p(X \leq 3)$. We have:

$$\begin{aligned} p(X \leq 3) &= p(X = 0) + p(X = 1) + p(X = 2) + p(X = 3) \\ &= 0.133 + 0.311 + 0.311 + 0.173 \\ &= .928 \end{aligned}$$

where we used the tables to find the probabilities. □