

# Some probability questions

Nikos Apostolakis

## 1 Coins

1. We toss a fair coin twice. What is the probability that:
  - (a) We get heads then tails.
  - (b) We get two tails.
  - (c) The first toss turns up heads.
  - (d) We get exactly one head.
  - (e) We get two heads.
2. We toss a fair coin three times. What is the probability that:
  - (a) We get exactly one head.
  - (b) We get exactly two heads.
  - (c) We get at most one head.
  - (d) We get at least two heads.
3. We now toss a biased coin: for this coin the probability that it will show tails is 0.7. We toss the coin twice. Find the probability of the following events:
  - (a) We get no heads.
  - (b) We get exactly one head.
  - (c) We get two heads.

## 2 Dice

For the following questions assume that all dice involved are fair, so it has equal probability of landing on each of its faces.

1. A fair dice is cast. What's the probability that:
  - (a) The result is four.
  - (b) The result is at most four.
  - (c) The result is three or less.
  - (d) The result is even.
  - (e) The result is odd.
  - (f) The result is a prime number.
2. Two fair dice are cast. What is the probability that the sum of the numbers in their top faces is:
  - (a) Six.
  - (b) Seven or eleven.
  - (c) More than seven.
  - (d) At least seven.
  - (e) Six or less.
  - (f) More than four and at most seven.
  - (g) Even.
  - (h) Odd.
  - (i) Prime.

### 3 Cards

For the following questions we assume that we are using a standard 52-card deck. Such a deck contains 52 cards that are divided into *kinds* and *suits*. There are 13 kinds

$A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$

and 4 suits, namely spades, clubs, diamonds, and hearts with symbols

♠, ♣, ♦, ♥

respectively. Each card has a kind and a suit, so for example we have the king of hearts ( $K♥$ ), the ace of spades ( $A♠$ ), the Jack of clubs ( $J♣$ ), the queen of diamonds ( $Q♦$ ), and the three of spades ( $3♠$ ). Additionally each suit has a color, clubs and spades are black while diamonds and hearts are red.

1. We draw a random card. What is the probability that this card is:
  - (a) The four of hearts.
  - (b) A four.
  - (c) A red four.
  - (d) A heart.
  - (e) A black card.
  - (f) A club.
  - (g) A red queen or a black ace.
  - (h) An ace or a diamond.
  - (i) A figure (that is a King, a Queen or a Jack).
  - (j) Not a figure.
  - (k) A card with value more than three but less than seven.
2. We draw two random cards *without replacement*, that is after we draw the first card we do not replace it in the deck. Find the probability of the following events:
  - (a) Both cards are red.
  - (b) At least one card is black.
  - (c) The first card is black and the second red.
  - (d) There is one red and one black card.
  - (e) Both cards are aces.
  - (f) Both cards have the same suit.
  - (g) Both cards are the same kind.
  - (h) There is an ace and a jack.
3. We draw two random cards *with replacement*, that is after we draw the first card (and examine it) we replace it in the deck and then we draw the second card (after reshuffling of course). Do parts (a) – (h) of the previous exercise.

## The answers

### 1 Coins

- $\frac{1}{4}$
  - $\frac{1}{4}$
  - $\frac{1}{2}$
  - $\frac{1}{2}$
  - $\frac{1}{4}$
- $\frac{3}{8}$
  - $\frac{3}{8}$
  - $\frac{1}{2}$
  - $\frac{1}{2}$
- 0.343
  - 0.441
  - 0.189

### 2 Dice

- $\frac{1}{6}$
  - $\frac{2}{3}$
  - $\frac{1}{2}$
  - $\frac{1}{2}$
  - $\frac{1}{2}$
  - $\frac{1}{2}$
- $\frac{5}{36}$
  - $\frac{2}{9}$
  - $\frac{5}{12}$
  - $\frac{7}{12}$
  - $\frac{5}{12}$

- (f)  $\frac{5}{12}$
- (g)  $\frac{1}{2}$
- (h)  $\frac{1}{2}$
- (i)  $\frac{15}{36}$

### 3 Cards

1. We draw a random card. What is the probability that this card is:

- (a)  $\frac{1}{52}$
  - (b)  $\frac{1}{13}$
  - (c)  $\frac{1}{26}$
  - (d)  $\frac{1}{4}$
  - (e)  $\frac{1}{2}$
  - (f)  $\frac{1}{4}$
  - (g)  $\frac{1}{13}$
  - (h)  $\frac{4}{13}$
  - (i)  $\frac{3}{13}$
  - (j)  $\frac{10}{13}$
  - (k)  $\frac{3}{13}$
2. (a)  $\frac{25}{102}$
- (b)  $\frac{77}{102}$
  - (c)  $\frac{13}{51}$
  - (d)  $\frac{26}{51}$
  - (e)  $\frac{1}{221}$
  - (f)  $\frac{4}{17}$
  - (g)  $\frac{1}{17}$
  - (h)  $\frac{8}{663}$

3. (a)  $\frac{1}{4}$   
(b)  $\frac{1}{2}$   
(c)  $\frac{1}{4}$   
(d)  $\frac{1}{2}$   
(e)  $\frac{1}{169}$   
(f)  $\frac{1}{4}$   
(g)  $\frac{1}{13}$   
(h)  $\frac{1}{169}$