Second Review for Math 05 Spring 2006

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Please note: You should fully justify your answers.

1. Find the equation of the line that:
   (a) passes through the point $(3, -2)$ and has slope equal to -2.
   (b) passes through the points $(2, 1)$ and $(-2, -11)$.
   (c) passes trough $(3, 4)$ and $(-7, 4)$.
   (d) passes through the points $(-1, 5)$ and $(-1, 6)$
   (e) is parallel to the line $2x - 4y = 5$ and has the same y-intercept as the line $5x - 3y = 6$.
   (f) is perpendicular to the line $y = -3x + 5$ and passes through $(3, 7)$.
   (g) is perpendicular to x-axis and passes through the point $(7, 11)$.
   (h) it has the same x-intercept as $2x + 3y = 12$ and the same y-intercept as $y = -3x + 6$.

2. Graph each of the lines in Question 1.

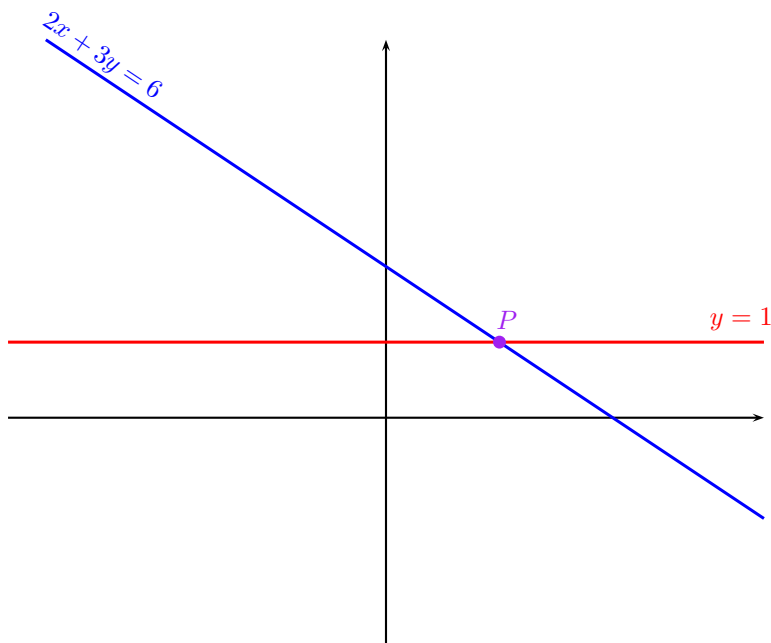3. Find the coordinates of the point P in the Figure 1.

Figure 1: Two intersecting lines

4. Solve for x and y:
$$\begin{cases} 2x - 3y & = -23 \\ 5x + 2y & = 9 \end{cases}$$

5. Solve the following system:
$$\begin{cases} 2x + 3y & = 16 \\ y & = -2x + 4 \end{cases}$$

6. Graph each of the following lines on the space provided in Figure 2:
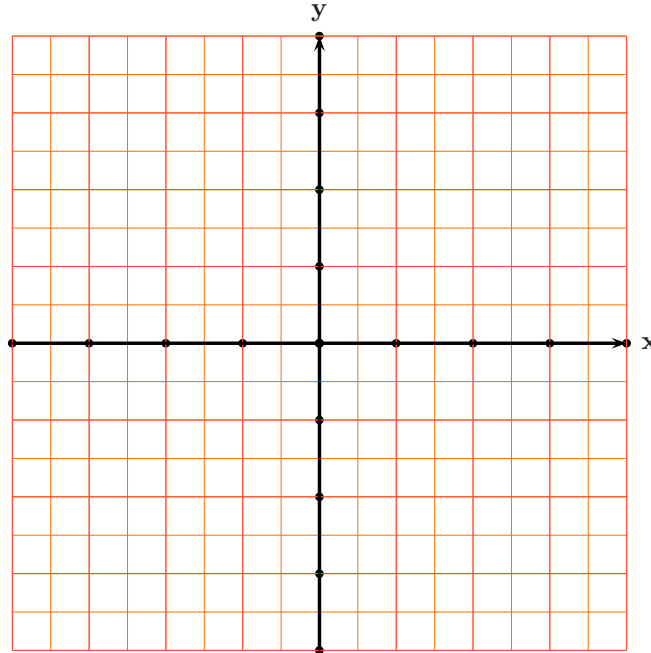
    (a) $x - 4y = -10$

    (b) $-3x + 2y = -12$

Figure 2: Graph paper for Question 6

7. Refer to the lines you graphed in Question 6.

    (a) (100 points) Explain using only the equations (i.e. you are not allowed to use the graph) why the two lines intersect in exactly one point.

    (b) Find the co-ordinates of the intersection *without using the graph*.

8. A chemist wants to combine a 30% alcohol solution with a 60% alcohol solution to form $600\,\text{ml}$ of a 50% solution. How much of each solution should be used to form the mixture?

9. John wants to invest $8000 in two different plans. Plan A has an annual interest rate of 5% and plan B has an interest rate of 3%. How much should he invest on each plan if he wants at the end of the year to gain interest of $332?

10. A boat makes a downstream trip of $36\,\text{mi}$ in $2\,\text{h}$. Returning upstream, it took $3\,\text{h}$ to make the trip. How fast can the boat travel in still water? What was the rate of the stream?

11. Simplify: $\left(\dfrac{3a^4b^9}{2a^2b^7}\right)\left(\dfrac{a^6b^3}{a^3b^2}\right)^2$.

12. Simplify: $\left(\dfrac{2xy^2}{z}\right)^2 (-xy^2z)^3$.

13. Simplify: $\dfrac{3x^5 - 2x^3 + 4x^2}{x^2}$.

14. A rectangle has sides of $3x - 7$ and $5x + 8$. Find its perimeter and its area.

15. Divide $8x^3 - 6x^2 + 2x$ by $4x + 1$.

16. The area of a rectangle is given by the polynomial $5x^4 - 2x^3 + 7x^2 + 5x - 2$. If one of the sides of the rectangle is equal to x^2, find the other side.

17. Factor each of the following polynomials completely. If you think that a polynomial is not factorable state so and explain why.

    (a) $35 - 42m - 18mn + 15n$

    (b) $4x^2 - 100$

    (c) $x^2 + 5x - 14$

    (d) $6x^2 + 11x - 10$

    (e) $x^3 + 18x - 9x^2$

    (f) $8x^3 - 27$

    (g) $x^3 + 125$

    (h) $x^2 + 4$

    (i) $12x^2 - 16x + 5$

    (j) $x^3 - 5x^2 - x + 5$

    (k) $x^2 + 6x + 13$

    (l) $7x^3 - 63x$

    (m) $7x^2 - 16x + 4$

    (n) $5x^7y^5 - 20x^5y$

    (o) $x^3y^3 - 2y^3 - 3x^3 + 6$

    (p) $3a^4b^2 - 9a^3b^3 - 12a^2b^4$.

18. If a ball is thrown upward from the roof of a 105-meter building with initial velocity of $20\,\text{m/s}$, it approximate height h after t seconds is given by

$$h = -5t^2 + 20t + 105\,.$$

    (a) How long will it take the ball reach the ground?

    (b) When will the ball be at a height of $80\,\text{m}$?

19. A paper box is to be made with a piece of cardboard $10\,\text{in}$ wide and $20\,\text{in}$ long. The box will be formed by cutting squares of equal size out of each of the four corners and folding up the sides to make a box. If the size of the cut out square is x find the volume of the box.
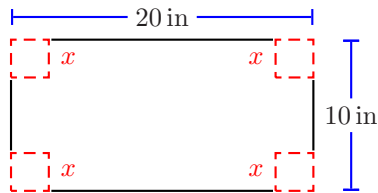

Figure 3: How to make a cardboard box.

20. One integer is 5 less than three times another integer. Find the two integers if their product is 50.

21. Solve $y^4 - 13y^2 + 36 = 0$.

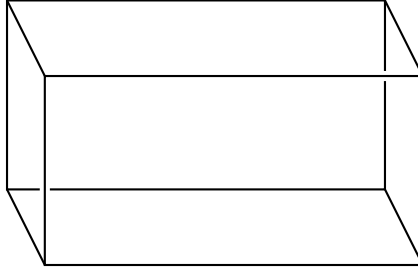22. The volume of a rectangular parallelepiped is given by $3x^3 + 10x^2 - 23x + 10$.

Figure 4: A rectangular parallelepiped

(a) Find the height of of the parallelepiped if the base has area $3x^2 + 13x - 10$.

(b) Find the volume if the area of the base is is 28.