Review Questions for the Second Exam

Fall 20017
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1. The following data are based on a survey taken by a consumer research firm. In this table x stands for the number of televisions in a household and p stands for the percentages of U.S. households with that many television sets.

x	0	1	2	3	4	5 or more
p	3%	11%		39%	12%	7%

   (a) Complete the missing percentage.

   (b) What is the probability that a household selected at random has fewer than three televisions?

   (c) What is the probability that a household selected at random has more than four televisions?

   (d) Compute the expected value of the x distribution (round televisions of 5 or more to 5).

   (e) Compute the standard deviation of the x distribution (round televisions of 5 or more to 5).

   (f) Draw the histogram of this probability distribution.

2. About 45% of those called for jury duty will find an excuse to avoid it. Suppose 8 people are randomly called for jury duty.

   (a) Using the appropriate table, fill in the following chart:

r	0	1	2	3	4	5	6	7	8
$P(r)$									

   (b) Find the expected value μ and the standard deviation σ of this probability distribution.

   (c) Draw the histogram of this probability distribution.

   (d) Determine the probability that all 8 serve on jury duty.

3. At a carnival you pay \$2.50 to play the following game: You draw four cards, with replacement, out of an ordinary deck. For each heart you get you are paid a dollar. Is the carnival expected to make or lose money from this game? What are the expected earnings or loses if a thousand people play this game?

4. A coin is tossed 30 times and lands with heads up 25 times. Should we suspect that this is not a fair coin?

   Hint. Calculate whether this result is within the "usual range" of successes of an appropriate binomial distribution.

5. Suppose x is a normally distributed variable with $\mu = 10$ and $\sigma = 1.8$. What is the probability that a randomly selected value of x will be between 9 and 11?

6. Suppose x is a normally distributed variable with $\mu = 3.1$ and $\sigma = 0.66$. Find an x-value so that the area under the normal curve to the left of x is 22% of the total area.

7. The scores of a certain College Entrance Examination are normally distributed with mean 510 and standard deviation 60.

   (a) A college will consider for admission only students that scored 650 or above in the College Entrance Examination. What percentage of students will be considered?

   (b) Find the three quartiles for the scores of this examination.

(c) Jose scored 630 in this examination. What is Jose's percentile?

(d) Stephanie's score was at the 89-th percentile. What was Stephanie's score?

8. Let x be a random variable that represents the level of glucose in the blood (milligrams per deciliter of blood) after a 12 hour fast. Assume that for people under 50 years old, x has a normal distribution with mean $\mu = 80.3$ and $\sigma = 20.2$.

(a) What is the probability that on a single test, $x < 40.1$?

(b) Suppose a doctor uses the average $\bar{x}$ for $n = 3$ tests, taken a week apart.
   i. What type of distribution does $\bar{x}$ have?
   ii. What is the probability that $65 < \bar{x} < 84.3$?

9. For a normal curve with mean μ and standard deviation σ, use the **empirical rule** to determine the percentage of area that lies to the **right** of $\mu + \sigma$? Sketch the curve and shade the appropriate region.

10. A new muscle relaxant appears on the market. Researchers from the firm that developed the product have done studies indicating that the time lapse between administration of the drug and beginning effects of the drug can be represented by a normally distributed variable x, with mean $\mu = 38$ minutes and standard deviation $\sigma = 5$ minutes.

(a) The drug is administered to one patient selected at random. Find the probability that the lapse for the drug to take effect is more than 41 minutes.

(b) The drug is administered to a random sample of 10 patients. Find the probability that the average time lapse for the 10 patients is more than 41 minutes.

(c) Why are the two results different? Is your result expected? Comment on the difference by referring to the Central Limit Theorem.

11. The numerical population of grade point averages at a college has mean 2.61 and standard deviation 0.5. If a random sample of size 100 is taken from the population, what is the probability that the sample mean will be between 2.51 and 2.71?

12. The batteries manufactured by a certain automobile battery manufacturer have a lifetime that is approximately normally distributed with a mean of 50 months and a standard deviation of 6 months.

(a) Find the probability that a randomly selected battery of this type will last less than 48 months.

(b) The company wants to advertise that they will fully refund any customer that buys a battery that lasts less than their guarantee period. What should the guarantee period be, if the company doesn't want to refund more than 15% of batteries sold?

(c) Find the probability that the mean of a random sample of 25 such batteries will be less than 48 months.

13. The lengths of time taken by students on an algebra proficiency exam (if not forced to stop before completing it) are normally distributed with mean 28 minutes and standard deviation 1.5 minutes.

(a) Find the proportion of students who will finish the exam if a 30-minute time limit is set.

(b) What should the time limit be if we want 97.5% of the students to finish in time?

(c) Six students are taking the exam today. Find the probability that all six will finish the exam within a 30-minute limit, assuming that times taken by students are independent.
   Hint. There is a binomial random variable here, whose value of p comes from part (a).

14. In the CEAFE exam there are 25 multiple choice questions each with four choices, and each question is worth 4 points. If a student select answers at random, find the probability that they will get

(a) a score of 60 or more.

(b) a score of 72 or more.

(c) a score of 76 or more.

15. A recent survey found that 7% of adults leaving in USA believe that chocolate milk comes from brown cows. Assume that a simple random sample of 300 adults leaving in USA is selected, and let r stand for the number of individuals among them that believe that chocolate milk comes from brown cows.

(a) What is the expected value of r?

(b) What is the standard deviation of r?

(c) Explain why the normal approximation to the binomial distribution applies to this example.

(d) Use a normal approximation to the binomial distribution to estimate the probability that between 15 and 27 people among the 300 selected believe that chocolate milk comes from brown cows.

16. Suppose that 2% of all cell phone connections by a certain provider are dropped. Find the probability that in a random sample of $1,500$ calls at most 40 will be dropped. First verify that the sample is sufficiently large to use the normal distribution.