Second Exam for MTH 23

November 7, 2017
Nikos Apostolakis

Name: Answers

Instructions:

This exam contains 7 pages (including this cover page) and 5 questions. Each question is worth 20 points, and so the perfect score in this exam is 100 points. Check to see if any pages are missing. Enter your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use only the provided formulae sheet. You may *not* use your book or notes.

You are allowed to use a calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- You have to enter the answer of each question in the provided box or blank line. You have to circle your answer in the multiple choice questions.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or other work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the last page; clearly indicate when you have done this.

1. The National Infomercial Marketing Association conducted a survey among *buyers* of a particular product, and the results are shown in the table below, where x represent the number of times buyers of a product had watched a TV infomercial before purchasing the product, and $p(x)$ the probability that buyers purchase the product after watching it x times. Values of 5 or more for x were treated as 5.

x	$p(x)$	$x\,p(x)$	x^2	$x^2\,p(x)$
1	0.27	0.27	1	0.27
2	0.41	0.82	4	1.64
3	0.08	0.24	9	0.72
4	0.09	0.36	16	1.44
5	0.15	0.75	25	3.57
Σ	1	2.44		7.82

(a) Complete the missing probability. The sum of all the probabilities $\Sigma p(x) = 1$. If we sum all the given probabilities we get: $0.27 + 0.08 + 0.09 + 0.15 = 0.59$. So $p(2) = 1 - 0.59 = \boxed{0.41}$

(b) What is the probability that buyers watch three or fewer infomercials before purchasing the product?

$$P(x \le 3) = p(1) + p(2) + p(3)$$
$$= 0.27 + 0.41 + 0.08$$
$$= 0.76$$

Answer: The probability is $\boxed{0.76}$

(c) Compute the expected value of this distribution.

See table above.

$$\mu = \Sigma\, x\,p(x) = 2.44$$

Answer: The expected value is $\mu = \boxed{2.44}$

Note: It's a good practice to pospone rounding until the final calculation

(d) Compute the standard deviation of this distribution.

$$\sigma = \sqrt{\Sigma x^2 p(x) - \mu^2} = \sqrt{7.82 - 2.44^2} = \sqrt{7.82 - 5.9536} = \sqrt{1.8664} \approx 1.37$$

Answer: The standard deviation is $\sigma = \boxed{1.37}$

2. About 45% of those called for jury duty will find an excuse to avoid it. Suppose 5 people are randomly called for jury duty, and let r stand for the number of people that actually show up, i.e. they do *not* find an excuse to avoid it.

This is a binomial distribution with $n=5$ and succes showing up, so $p=1-.45=.55$

(a) Using the appropriate table, fill in the following chart:

r	0	1	2	3	4	5
$P(r)$	0.19	0.113	.276	.337	.206	.050

(b) Find the expected value μ and the standard deviation σ of this probability distribution.

We could use the table but we don't need to!

This is a Binomial Distribution and we have formulae: $n=5$, $p=.55$, $q=.45$

$\mu = np = 5 \cdot 0.55 = 2.75$

$\sigma = \sqrt{npq} = \sqrt{5 \cdot 0.55 \cdot 0.45} = \sqrt{1.2375} \approx 1.12$

Answer: The expected value is $\mu = \boxed{2.75}$

The standard deviation is $\sigma = \boxed{1.12}$

(c) Determine the probability that all 5 serve on jury duty.

From the

$P(5) = 0.05$

Second way: Use the formula $P(r) = \binom{n}{r} p^r q^{n-r}$

$P(5) = \binom{5}{5} (.55)^5 (.45)^{5-5}$

$= 1 \cdot (\quad) (.45)^0$

$\approx 1 \cdot (0.5032) \cdot 1$

≈ 0.5032

Answer: The probability is $\boxed{0.05}$

3. The Research Department of a company that manufactures watches has determined that their watches have an average lifetime of 28 months before certain electronic components deteriorate, causing the watches to become unreliable. They have also find that the standard deviation of lifetimes is 5 months and that the distribution of lifetimes is normal.

   (a) If the company guarantees a full refund on any defective watch for 2 years after purchase, what percentage of the total production should the company expect to replace?

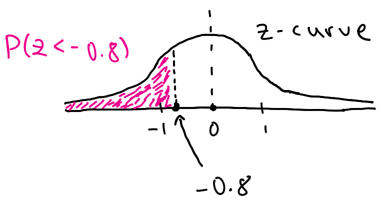
Let the random variable x stand for the lifetime of a watch.

2 years is 24 months so we want $P(x < 24)$

Since x is normal we will use the z-tables

The z-score that corresponds to $x = 24$ is

$$z = \frac{x-\mu}{\sigma} = \frac{24-28}{5} = -\frac{4}{5} = -0.8$$



From the table

So $P(x < 24) = P(z < -0.8) = 0.2119$ wich is 21.19%

Answer: The ~~probability~~ percentage is $\boxed{21.19\%}$

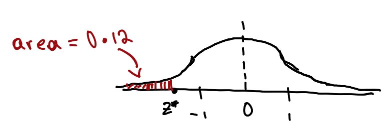
   (b) If the company doesn't want to make refunds on more than 12% of the watches it makes, how long should the guarantee period be (to the nearest month)?

We are looking for a value x^* so that $P(x < x^*) = .12$

We look inside the body of the z table for .12 and we find that it is exactly in the middle between 12.10 that corresponds to $z = -1.17$, an 11.90 that corresponds to $z = -1.18$. So we take $z^* = \frac{-1.17 + (-1.18)}{2} = -1.175$

The corresponding raw score is

$$x^* = \sigma \cdot z^* + \mu = 5 \cdot (-1.175) + 28 = 22.125$$



Rounding we get 22 months

Answer: The guarantee should be $\boxed{22}$ months.

4. Let x be a random variable that represents the level of glucose in the blood (milligrams per deciliter of blood) after a12 hour fast. The random variable x has a distribution that is approximately normal with $\mu = 85$ and $\sigma = 20$.

   (a) What is the probability that x is more than 60?

   We want $P(x > 60)$

   The z-score that corresponds to $x = 60$ is $z = \frac{x - \mu}{\sigma} = \frac{60-85}{20} = -\frac{25}{20} = -1.25$

   So $P(x > 60) = P(z > -1.25) = P(z < 1.25) = 0.8944$

   could also have used $1 - P(z < -1.25)$

   Answer: The probability is $\boxed{0.8944}$

   (b) Suppose that a doctor uses the average $\bar{x}$ for a sample of $n = 4$ tests, taken a week apart.

   i. What type of distribution does $\bar{x}$ have?

   Since the population x is normal by CLT we have that $\bar{x}$ is normal with $\mu_{\bar{x}} = \mu_x = 85$ and $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{20}{\sqrt{4}} = \frac{20}{2} = 10$

   Answer: $\bar{x}$ follows a normal distribution, with $\mu =$ 85 and $\sigma =$ 10

   ii. What is the probability that $75 < \bar{x} < 100$?

   The z-score for $\bar{x} = 75$ is $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{75-85}{10} = -\frac{10}{10} = -1$

   The z-score for $\bar{x} = 100$ is $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{100-85}{10} = \frac{15}{10} = 1.5$

   So $P(75 < \bar{x} < 100) = P(-1 < z < 1.5)$

   $= P(z < 1.5) - P(z < -1)$
   $= 0.9332 - 0.1587$
   $= 0.7745$

   Answer: The probability is $\boxed{0.7745}$

5. Long experience with a certain course shows that about 71% of the students pass. This fall 80 students are taking this course. Let r be a random variable that represents the number of students that will pass.

   (a) What is the number of students expected to pass this semester?

   r follows a normal distribution with $n = 80$ and $p = 0.71$

   So the expected value is $\mu = np$
   $= 80 \cdot 0.71$
   $= 56.8$

   Rounding to the nearest integer gives 57.

   Answer: The number of students expected to pass this semester is $\boxed{57}$

   (b) Explain why the normal approximation to the binomial would apply.

   $q = 1 - .71 = .29$

   $np = 56.7 > 5$
   $nq = 80 \cdot 0.29 = 23.275$
   $\Rightarrow$ Since both np and nq are more than five the normal approximation to binomial applies.

   (c) Estimate the probability of at least 60 students passing.

   $\mu_r = 56.8$
   $\sigma_r = \sqrt{npq}$
   $= \sqrt{80 \cdot 0.71 \cdot 0.29}$
   $= \sqrt{16.472}$
   ≈ 4.06

   So r is approximated by a normal random variable x with $\mu = 56.8$ and $\sigma = 4.06$ So

   $P(r \geq 60) \approx P(x > 60.5) = P(z > 0.93)$

   $z = \frac{60.5 - 56.8}{4.06} = \frac{3.7}{4} = 0.925 \approx 0.93$

   $= P(z < -0.93)$
   $= .1762$
   $\approx .18$

   Answer: The probability is $\boxed{0.18}$

Useful Formulae

Discrete random variables:

$$\mu = \sum x\, p(x) \qquad \sigma = \sqrt{\sum (x-\mu)^2\, p(x)} \quad \boxed{= \sqrt{\sum x^2\, p(x) - \mu^2}}$$

Binomial Distribution

$$q = 1-p \qquad P(r) = \binom{n}{r} p^r\, q^{n-r} \qquad \mu = n\,p \qquad \sigma = \sqrt{n\,p\,q}$$

$$n! = 1\cdot 2\cdot 3\cdots n \qquad \binom{n}{r} = \frac{\overbrace{n(n-1)\cdots(n-r+1)}^{r \text{ factors}}}{r!}$$

Normal Distribution:

$$z = \frac{x-\mu}{\sigma} \qquad x = z\,\sigma + \mu$$

$$P(a < z < b) = P(z < b) - P(z < a)$$

$$P(z > a) = 1 - P(z < a) = P(z < -a)$$

Sampling distribution:

$$\mu_{\bar{x}} = \mu_x \qquad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

$\bar{x}$ is normal if x is. It is approximately normal if $n \geq 30$.

Normal Approximation to the Binomial:

Valid when $n\,p > 5$ **and** $n\,q > 5$.

$$P(k \leq r \leq l) \approx P(k - 0.5 \leq x \leq l + 0.5)$$