First Exam for MTH 23

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Name: ____

Instructions:

This exam contains 6 pages (including this cover page) and 5 questions. Each question is worth 20 points, and so the perfect score in this exam is 100 points. Check to see if any pages are missing. Enter your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use only the provided formulae sheet. You may *not* use your book or notes.

You are allowed to use a calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- You have to enter the answer of each question in the provided box or blank line. You have to circle your answer in the multiple choice questions.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or other work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the last page; clearly indicate when you have done this.

1. Consider the following set of data:

$$23 \quad 15 \quad 27 \quad 15 \quad 30$$

   (a) Find the median. **Answer:** The median is ☐

   (b) Find the mode. **Answer:** The mode is ☐

   (c) Find the sample mean. **Answer:** $\bar{x} =$ ☐

   (d) Find the sample standard deviation. **Answer:** $s =$ ☐

2. A random sample of the percentage of people in 60 different counties, that voted for a certain party in the elections is given below.

31	33	34	34	35	35	36	38	38	38
39	40	40	40	40	41	41	41	41	41
41	41	42	42	43	44	44	44	45	45
46	46	46	46	47	48	49	49	49	49
50	51	52	52	53	53	53	53	53	55
56	56	57	57	59	62	66	66	66	68

Given that the mean is $\bar{x} = 46.15$ and the standard deviation is $s = 8.63$:

   (a) Find a 75% Chebyshev interval about the mean for the data set above.

   Answer: The 75% Chebyshev interval is ☐

   (b) How many data values does Chebyshev's theorem predict will be within two standard deviations of the mean?

   Answer: ☐ values.

   (c) How many of the data values are within two standard deviations of the mean?

   Answer: ☐ values.

   How does this compare to your result in Part (b)?

3. Let the random variable x represents percentage change in neighborhood population in the last few years, and the random variable y the crime rate in the same neighborhood (measured in crimes per $1,000$ people). A random sample of six neighborhoods in a city gave the following table of paired data:

x	29	2	11	17	7	6
y	173	35	132	127	69	53

Given that $\sum x = 72$, $\sum y = 589$, $\sum x^2 = 1340$, $\sum y^2 = 72,277$, and $\sum xy = 9,499$:

(a) Compute the coefficient of correlation r.

Answer: $r =$ ☐

(b) Find the equation of the least square line.

Answer: The equation is ☐

(c) Use the equation from Part (b) to estimate the crime rate for a neighborhood with 12% change in population.

Answer: The crime rate is estimated to be ☐

4. Consider the experiment of rolling two dice. The following table lists all possible outcomes.

1 6	2 6	3 6	4 6	5 6	6 6
1 5	2 5	3 5	4 5	5 5	6 5
1 4	2 4	3 4	4 4	5 4	6 4
1 3	2 3	3 3	4 3	5 3	6 3
1 2	2 2	3 2	4 2	5 2	6 2
1 1	2 1	3 1	4 1	5 1	6 1

Find the probability that the sum of the outcomes of the two dice is less or equal to 5.

Answer: The probability is

5. In a sales effectiveness seminar, a group of sales representatives tried two approaches to selling a customer a new car. The results are summarized in the table below:

	Sale	**No Sale**	**TOTAL**
Aggressive	270	310	580
Passive	416	164	580
TOTAL	686	474	1160

Compute the following probabilities:

(a) Find the probability that a customer selected at random bought a new car.

Answer: The probability is ☐

(b) Find the probability that a customer selected at random will buy a new car given that an aggressive sales approach is used.

Answer: The probability is ☐

(c) Find the probability that a customer selected at random will buy a car **and** that an aggressive sales approach was used.

Answer: The probability is ☐

Useful Formulae

Mean: $\bar{x} = \dfrac{\sum x}{n}, \qquad \mu = \dfrac{\sum x}{N}$

Standard Deviation: $s = \sqrt{\dfrac{\sum x^2 - \frac{1}{n}\left(\sum x\right)^2}{n-1}}, \qquad \sigma = \sqrt{\dfrac{\sum x^2 - \frac{1}{N}\left(\sum x\right)^2}{N}}$

Correlation coefficient: $r = \dfrac{\mathrm{SS}_{xy}}{\sqrt{\mathrm{SS}_{xx} \cdot \mathrm{SS}_{yy}}}$, where:

$$\mathrm{SS}_{xx} = \sum x^2 - \frac{1}{n}\left(\sum x\right)^2$$

$$\mathrm{SS}_{xy} = \sum x\, y - \frac{1}{n}\left(\sum x\right)\left(\sum y\right)$$

$$\mathrm{SS}_{yy} = \sum y^2 - \frac{1}{n}\left(\sum y\right)^2$$

Least Squares Regression Line: $\hat{y} = b\,x + a$, where:

$$b = \frac{\mathrm{SS}_{\mathrm{xy}}}{\mathrm{SS}_{\mathrm{xx}}}, \qquad a = \bar{y} - b\,\bar{x}$$