Take Home Make up for First Exam for MTH 23

October 12, 2017

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Due Date: Tuesday October 17, 2017

1. A random sample of 30 heights (in hundredths of inches, $100 = 1$ inch) from a population is given below:

Min value = 6348

Max value = 7151

Range = 7151 − 6348
= 803

6578	$\boxed{7151}$	6939	6821	6778
6869	6980	7001	6790	6678
6648	6762	6830	6711	6827
7109	6646	6864	7123	6713
6783	6887	$\boxed{6348}$	6842	6762
6720	7084	6749	6653	6544

$\frac{803}{5} = 160.6$

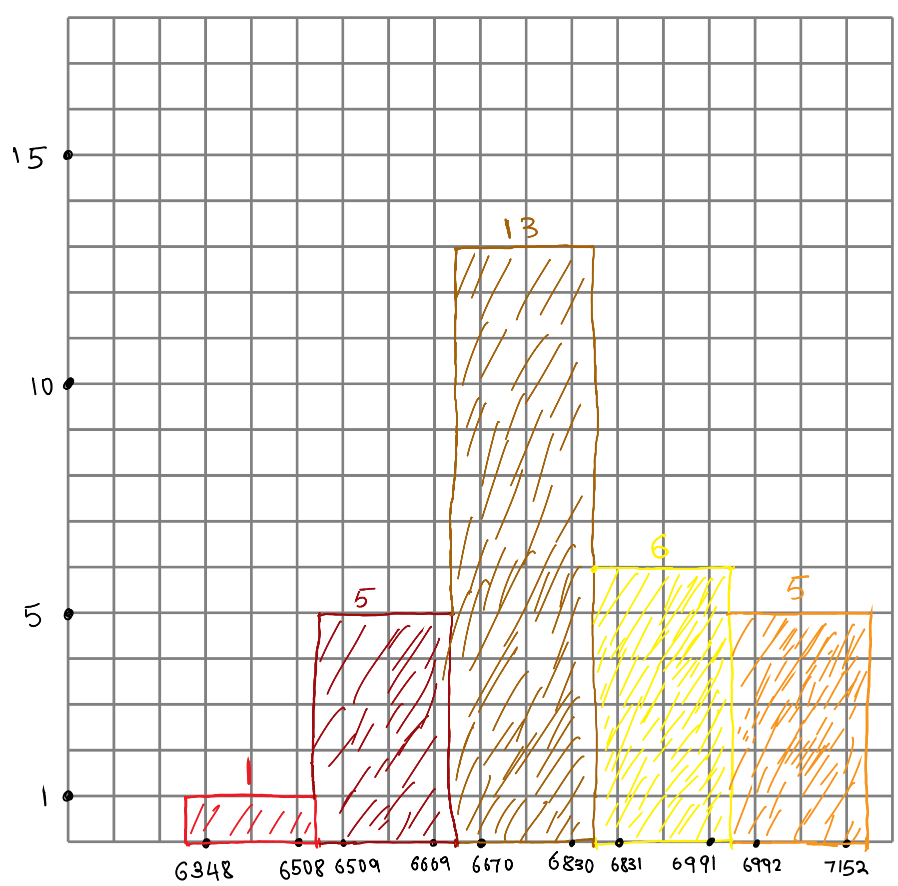
So class width = 161

(a) Construct a frequency table for the above data, listing the class limits, the class boundaries, the class midpoint, the frequency and the relative frequency. Use five classes.

(b) Draw a histogram for the frequency table in Part (a).

You can use the following grid:

Class Limits	Class Boundaries	Tally	Frequency
6348 − 6508	6347.5 − 6508.5		1
6509 − 6669	6508.5 − 6670.5		5
6670 − 6830	6670.5 − 6830.5		13
6831 − 6991	6830.5 − 6991.5		6
6992 − 7152	6991.5 − 7152.5		5
			30



2. For the following data

47 59 50 56 56 51 53 57 52 49

calculate:

(a) The sample mean.

(b) The sample standard deviation.

(c) The range.

(d) The median.

(e) The mode

(f) The first and third quartiles.

3. Let x be the age of a bighorn sheep (in years) and y the mortality rate (percent that dies) for that age group. So for example, if $x = 1$, then $y = 14.0$ and that means that 14% of bighorn sheep between 1 and 2 years old died. A random sample of Arizona bighorn sheep gave the following information:

x	1	2	3	4	5
y	14.0	18.9	14.4	19.6	20.0

(a) Draw a scatter diagram. See the grid.

(b) Find the equation of the least square regression line and plot it in the same graph you used in part (a).

(c) Find the correlation coefficient r.

You can use the following grid

x	y	x^2	y^2	xy
1	14.0	1	196	14.0
2	18.9	4	357.21	37.8
3	14.4	9	207.36	43.2
4	19.6	16	384.16	78.4
5	20.0	25	400	100
Σ 15	86.9	55	1544.73	273.4

b) Least square line

$\hat{y} = a + bx$

$b = \frac{SS_{xy}}{SS_{xx}} = \frac{12.7}{10} = 1.27$

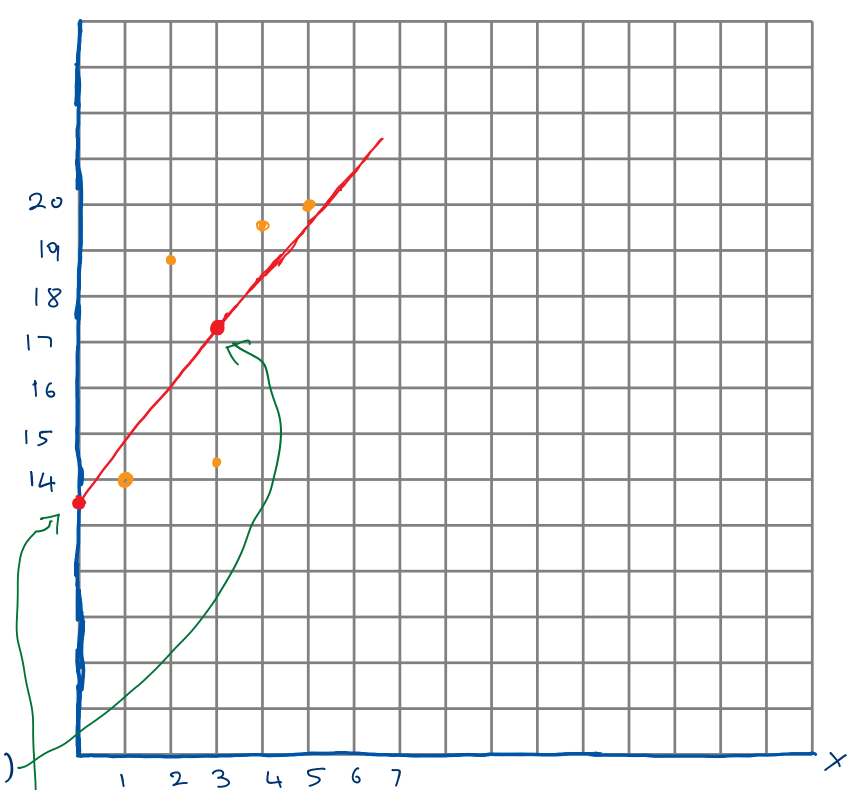
$a = \bar{y} - b\bar{x}$

$= 17.38 - 1.27 \cdot 3$

$= 13.57$

So regression line

$\boxed{\hat{y} = 13.57 + 1.27x}$



Graphing the l.s.l.

Two points $(\bar{x}, \bar{y}) = (3, 17.38)$

$(0, a) = (0, 13.57)$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 55 - \frac{15^2}{5}$

$= 55 - 45$

$= 10$

$SS_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 1544.73 - \frac{(81.9)^2}{5}$

$= 1544.73 - 7551.61$

$= 34.408$

$SS_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}$

$= 273.4 - \frac{15 \cdot 86.9}{5}$

$= 273.4 - 260.7$

$= 12.7$

$\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3$

$\bar{y} = \frac{\Sigma y}{n} = \frac{86.9}{5} = 17.38$

c) $r = \frac{SS_{xy}}{\sqrt{SS_{xx} \cdot SS_{yy}}} = \frac{12.7}{\sqrt{10 \cdot 34.408}} \approx \frac{12.7}{18.549} \approx 0.68$

Question 2

To answer the last 5 parts we sort the data:

middle   mode

47, 49, 50, 51, 52, | 53, 56, 56, 57, 59

c) The range is $59 - 47 = \underline{\underline{12}}$

d) There are 10 values so the median is the mean of the fifth and sixth value

so $\text{median} = \frac{52+53}{2} = \underline{\underline{52.5}}$

e) The most frequent value is 56. So the mode is $\underline{\underline{56}}$

f) The first quartile is the median of the first half of the data, that is the third value:

$Q_1 = 50$

The third quartile is the median of the second half of the data, that is the eigth value.

$Q_3 = 56$

For the first two parts we use the table:

	x	x^2
	47	2209
	49	2401
	50	2500
	51	2601
	52	2704
	53	2809
	56	3136
	56	3136
	57	3249
	59	3481
Σ	530	28226

a) $\bar{x} = \frac{\Sigma x}{n} = \frac{530}{10} = 53$

b) $$s^2 = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1} = \frac{28\,226 - \frac{280900}{10}}{9}$$

$$= \frac{28226 - 28090}{9}$$

$$= \frac{136}{9}$$

$$\approx 15.11$$

So $s \approx \sqrt{15.11} \approx 3.89$

4. Two cards are drawn from a standard 52-card deck, one after the other, *with replacement*, that is, after the first card is drawn we put it back, reshuffle, and then draw the other. Let A be the event "The first card is black", and B be the event "The second card is red".

   The outcome of the first event does not influence the second since the card is put back in

   (a) Are the event A and B independent?

   $\boxed{\text{A. Yes}}$ B. No

   $P(A) = \frac{26}{52} = 0.5$

   $P(B) = \frac{26}{52} = 0.5$

   (b) Find the probability $P(A$ **and** $B)$. $P(A \text{ and } B) = P(A) \cdot P(B) = .5 \cdot .5 = .25$

   (c) Find the probability $P(A$ **or** $B)$. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .5 + .5 - .25 = .75$

5. Two cards are drawn from a standard 52-card deck, one after the other, *without replacement*, that is, after the first card is drawn we put it aside and then draw the other. Let A be the event "The first card is black", and B be the event "The second card is red".

   The outcome of the first event influences the second. Depending on whether the first card was black or red there are more red or black cards.

   (a) Are the event A and B independent?

   A. Yes $\boxed{\text{B. No}}$

   (b) Find the probability $P(A$ **and** $B)$.

   (c) Find the probability $P(A$ **or** $B)$.

   } ANSWERS ON NEXT PAGE

6. The breakdown of the student body in a class according to race/ethnicity and gender is shown in the table below:

	White	Black	Hispanic	Asian	Other	Total
Male	25	12	6	3	1	47
Female	26	15	5	3	4	53
Total	51	27	11	6	5	100

A student is randomly selected from this class. (To select "randomly" means that every student has the same chance of being selected.) Find the probabilities of the following events:

(a) The selected student is Female. $P(\text{Female}) = \frac{53}{100} = 0.53$

(b) The selected student is Hispanic **or** Black. $P(\text{Hispanic OR Black}) = P(\text{Hisp}) + P(\text{Black}) = \frac{27}{100} + \frac{11}{100} = \frac{38}{100} = 0.38$

because we have mutually exclusive events

(c) The selected student is an Asian Male. $P(\text{Asian Male}) = \frac{3}{100} = 0.03$

Not mutually exclusive events

(d) The selected student is Asian **or** Male. $P(\text{Asian OR Male}) = P(\text{Asian}) + P(\text{Male}) - P(\text{Asian Male})$

$= \frac{6}{100} + \frac{47}{100} - \frac{3}{100} = \frac{50}{100} = 0.50$

(e) The selected student is **not** Other.

(f) The selected student is Black **given** that she is Female.

(g) The selected student is Female **given** that they are Black.

(h) The selected student is White **or** Male.

e) $P(\text{Other}) = \frac{5}{100} = 0.05$ So $P(\text{NOT Other}) = 1 - 0.05 = 0.95$

f) There are 53 Female students, 15 of them Black. So:

$P(\text{Black} \mid \text{Female}) = \frac{15}{53} \approx 0.283$

g) There are 27 Black students, 15 of them Black. So:

$P(\text{Female} \mid \text{Black}) = \frac{15}{27} \approx 0.56$

h) $P(\text{White OR Male}) = P(\text{White}) + P(\text{Male}) - P(\text{White Male})$

$= 0.51 + 0.47 - 0.25 = 0.73$

Question 5 In a standard deck there are 26 red and 26 black cards.

So $p(A) = \frac{26}{52} = 0.5$

b) Since A, B are not independent we use the formula:

$$P(A \text{ and } B) = p(A) \cdot p(B \text{ given } A)$$

Now $P(B \text{ given } A) = \frac{26}{51} \approx .51$ because if A happened there are 51 cards in the deck, and 26 of them are black.

So $P(A \text{ and } B) \approx .5 \cdot .51 = .255$

c) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

To find P(B) we use the formula $P(B) = P(A) \cdot P(B \text{ given } A) + P(\text{not} A) \cdot P(B \text{ given not } A)$

$P(B \text{ given not } A) = \frac{25}{51} \approx .49$

$= .5 \cdot .51 + .5 + .49$

$= .5$

So $P(A \text{ or } B) = .5 + .5 - .225 = .775$