First Exam for MTH 23

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Nikos Apostolakis

Name: ANSWERS

Instructions:

This exam contains 6 pages (including this cover page) and 4 questions. Each question is worth 25 points, and so the perfect score in this exam is 100 points. Check to see if any pages are missing. Enter your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use only the provided formulae sheet. You may *not* use your book or notes.

You are allowed to use a calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- You have to enter the answer of each question in the provided box or blank line. You have to circle your answer in the multiple choice questions.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or other work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the last page; clearly indicate when you have done this.

Data already sorted!

1. Consider the following set of data:

middle of the first half — middle of second half

50 53 59 59 $\boxed{63 \mid 63}$ 72 72 72 $\boxed{72 \mid 72}$ 76 78 81 $\boxed{83 \mid 84}$ 84 84 90 93

middle

$\frac{83+84}{2} = 83.5$

$n = 20$ So median is the mean of 10^{th} and 11^{th} value.

(a) Find the median. **Answer:** The median is $\boxed{72}$

(b) Find the mode. **Answer:** The mode is $\boxed{72}$

72 occurs 5 times
it's the most frequent value

(c) Find the sample mean. **Answer:** $\bar{x} = \boxed{73}$

(d) Find the sample standard deviation. **Answer:** $s = \boxed{12.09}$

(e) Find the first and third quartile. **Answer:** $Q_1 = \boxed{63}$

Q_1 is the median of the first 10 values. So the mean of 5^{th} and 6^{th} values.

Answer: $Q_3 = \boxed{83.5}$

Q_3 is the median of the last 10 values. So the mean of 15^{th} and 16^{th} values.

x	x^2
50	2500
53	2809
59	3481
59	3481
63	3969
63	3969
72	5184
72	5184
72	5184
72	5184
72	5184
76	5786
78	6084
81	6561
83	6869
84	7056
84	7056
84	7056
90	8100
93	8649
Σ 1460	109356

$$\bar{x} = \frac{1460}{20} = 73$$

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{109\,356 - \frac{(1460)^2}{20}}{19}}$$

$$= \sqrt{\frac{109\,356 - \frac{2131600}{20}}{19}}$$

$$= \sqrt{\frac{109\,356 - 106580}{19}}$$

$$= \sqrt{\frac{2776}{19}}$$

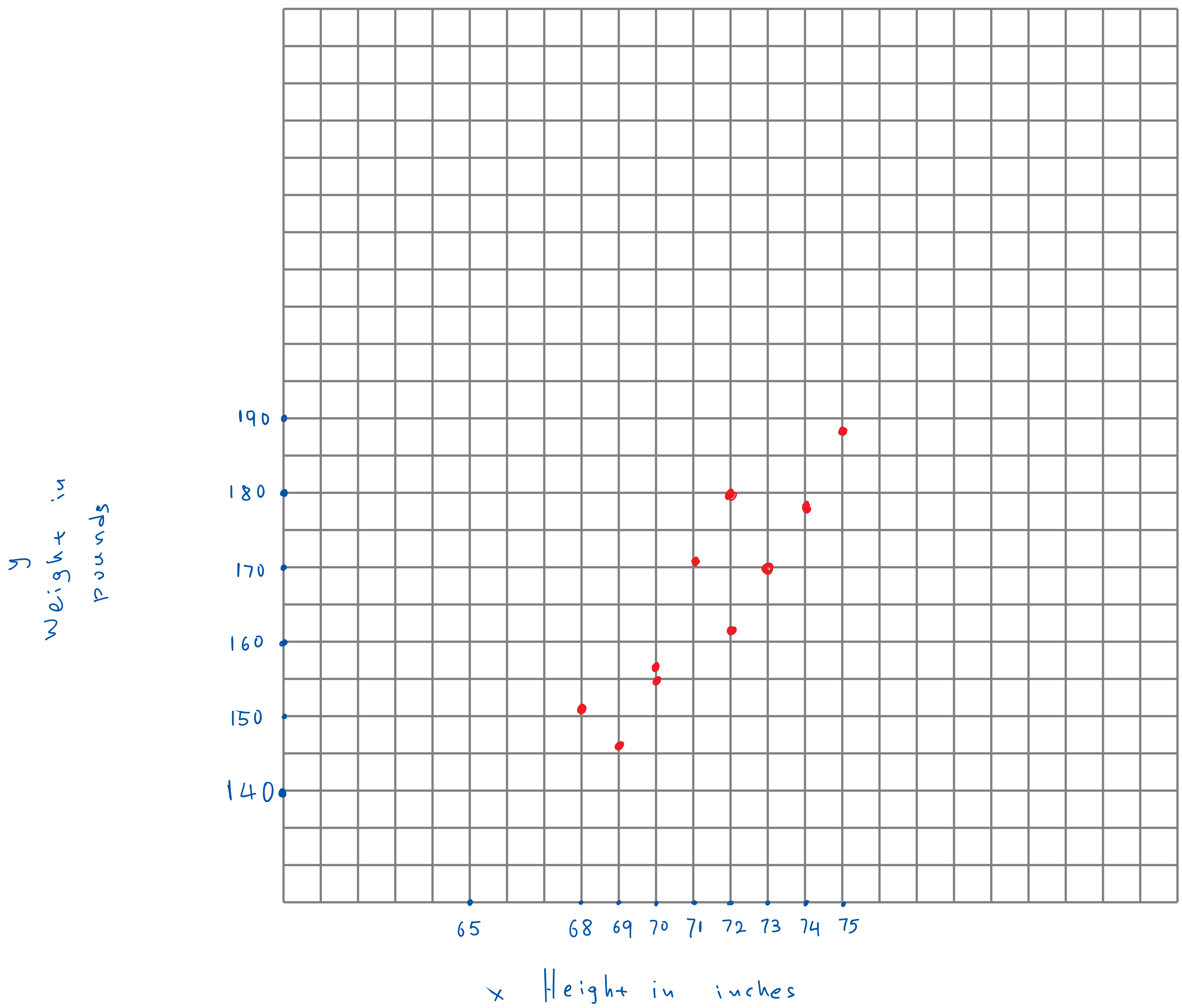
$$\approx \sqrt{146.105}$$

$$\approx 12.09$$

2. The height x in inches and the weight in pounds y for twelve men aged 25 is given in the table below:

x	68	72	69	72	70	73	70	73	71	74	72	75
y	151	163	146	180	157	170	164	175	171	178	160	188

(a) Plot a scatter diagram of the data. Remember to label your axes appropriately and choose a consistent scale.



(b) Based on a scatter diagram, would you estimate the correlation coefficient to be positive, close to zero, or negative?

Please circle one of the following choices:

A. Positive B. Close to zero C. Negative

(c) Interpret your results from parts (a) and (b).

There is a positive correlation between height and weight.

So we expect that the tallest a person the more they weigh.

3. Consider the experiment of rolling two dice. The following table lists all possible outcomes.

1 6	2 6	3 6	4 6	5 6	6 6
1 5	2 5	3 5	4 5	5 5	6 5
1 4	2 4	3 4	4 4	5 4	6 4
1 3	2 3	3 3	4 3	5 3	6 3
1 2	2 2	3 2	4 2	5 2	6 2
1 1	2 1	3 1	4 1	5 1	6 1

$1 + 3 + 5 + 5 + 3 + 1 = 18$

Find the probability that the sum of the outcomes of the two dice is even.

$\frac{18}{36} = \frac{1}{2} = .5$

Answer: The probability is $\boxed{0.5}$

4. In a sample of 902 individuals under 40 who were or had previously been married, each person was classified according to gender and age at first marriage. The results are summarized in the following table:

	Teenager	**Twenties**	**Thirties**	**TOTAL**
Male	43	293	114	450
Female	82	299	71	452
TOTAL	125	502	185	902

Suppose an individual is selected at random from that sample.

(a) Find the probability that the individual selected was a teenager at first marriage.

Out of 902 individuals 125 were teenagers at first marriage.

So $P = \frac{125}{902} \approx 0.14$

Answer: The probability is $\boxed{0.14}$

(b) Find the probability that the individual selected was a teenager at first marriage, given that the person is male.

Out of 450 males 43 were teenagers at first marriage.

So

$P(\text{Teenager} \mid \text{Male}) = \frac{43}{450} \approx 0.10$

Answer: The probability is $\boxed{0.10}$

(c) Determine whether or not the events F: female and E: was a teenager at first marriage are independent.

A. Yes, they are independent. $\boxed{\text{B. No, they are not independent.}}$

1st way:

$P(F) = \frac{452}{902} \approx 0.5$, $P(E) = 0.14$

$P(F \text{ and } E) = \frac{82}{902} \approx 0.09$

$P(F) \cdot P(E) \approx 0.5 \cdot 0.14 = 0.07$

So $P(F \text{ and } E) \neq P(F) \cdot P(E)$

2nd way

$P(E) = 0.14$

$P(E \mid F) = \frac{82}{452} \approx 0.18$

So since $P(E) \neq P(E \mid F)$

Useful Formulae

Mean: $\bar{x} = \dfrac{\sum x}{n}, \qquad \mu = \dfrac{\sum x}{N}$

Standard Deviation: $s = \sqrt{\dfrac{\sum x^2 - \frac{1}{n}\left(\sum x\right)^2}{n-1}}, \qquad \sigma = \sqrt{\dfrac{\sum x^2 - \frac{1}{N}\left(\sum x\right)^2}{N}}$

Correlation coefficient: $r = \dfrac{\mathrm{SS}_{xy}}{\sqrt{\mathrm{SS}_{xx} \cdot \mathrm{SS}_{yy}}}$, where:

$$\mathrm{SS}_{xx} = \sum x^2 - \frac{1}{n}\left(\sum x\right)^2$$

$$\mathrm{SS}_{xy} = \sum x\, y - \frac{1}{n}\left(\sum x\right)\left(\sum y\right)$$

$$\mathrm{SS}_{yy} = \sum y^2 - \frac{1}{n}\left(\sum y\right)^2$$

Least Squares Regression Line: $\hat{y} = b\,x + a$, where:

$$b = \frac{\mathrm{SS}_{\mathrm{xy}}}{\mathrm{SS}_{\mathrm{xx}}}, \qquad a = \bar{y} - b\,\bar{x}$$