

# Fourth Quiz for CSI35

Nikos Apostolakis

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**Directions:** This quiz is due Wednesday December 3, at 4:00 PM. Please make sure to *justify* all your answers.

1. A graph  $G$  has six vertices with the following degrees: 5, 3, 3, 2, 4, 1.
  - (a) How many edges does  $G$  have?
  - (b) Draw two non-isomorphic such graphs.
2. Are the two graphs in Figure 1 isomorphic? How about the graphs in Figure 2? or in Figure 3? Prove your answers.

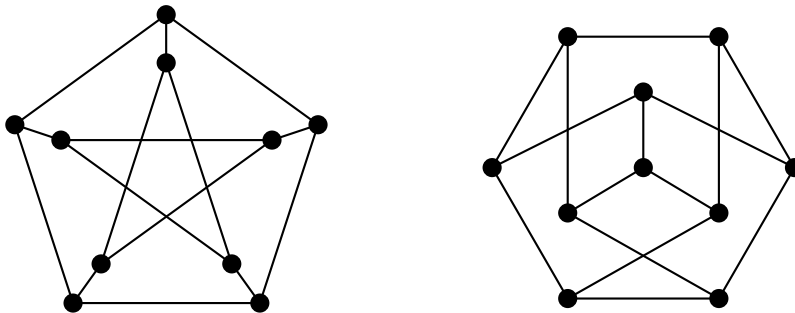


Figure 1: The first two graphs of Question 2

3. Let  $P_n$ ,  $C_n$ ,  $K_n$ ,  $W_n$ , and  $Q_n$  be the families of paths, cycles, complete graphs, wheels, and cubes respectively. For each of these families:
  - (a) Find a formula for the number of edges and prove your result.
  - (b) Determine the values of  $n$  for which the graph is bipartite.
  - (c) Determine the values of  $n$  for which the graph is Eulerian.
  - (d) Determine the values of  $n$  for which the graph is Hamiltonian.
4. Consider the graph  $P$  in left side of Figure 1.

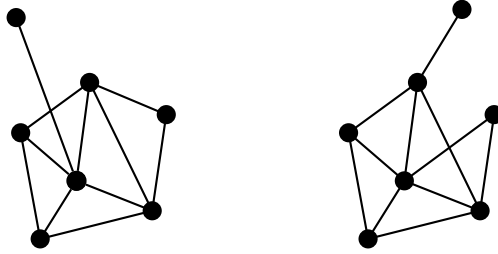


Figure 2: The middle two graphs of Question 2

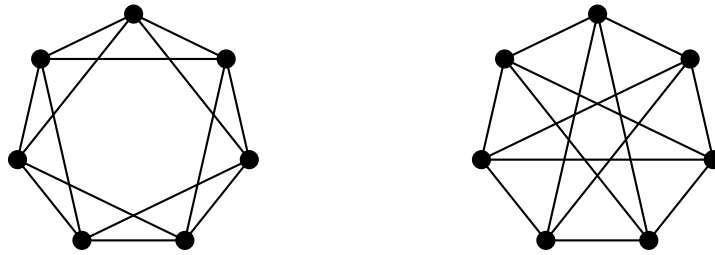


Figure 3: The last two graphs of Question 2

- (a) Find a Hamilton path in  $P$ .
  - (b) **Extra Credit:** Prove that  $P$  is not Hamiltonian.
5. Prove that the two graphs in Figure 4 are isomorphic.
  6. Prove that the graph in the left side of Figure 4 is not Hamiltonian.
  7. Count von Diamond has been murdered in his estate. The internationally known detective (and part time graph theorist) Inspector Clouseau has been called in to investigate. The butler claims that he saw the gardener enter the pool room (where the murder took place) and then, shortly after, leave that room by the same door. On the other hand, the gardener says that he cannot be the man that the butler saw because he entered the house, went through each door exactly once and then left the house. Inspector Clouseau checks the floor plan (see Figure 5) and within minutes declares the case solved. Who done it?
  8. **Extra Credit:** Is it possible to walk the seven bridges of Königsberg so that you cross every bridge exactly twice, once in every direction?

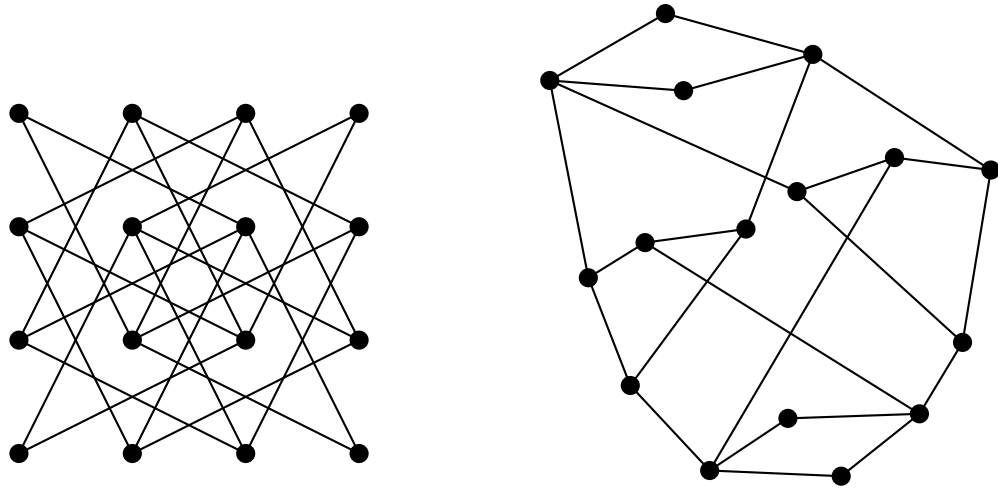


Figure 4: The graphs of Question 5

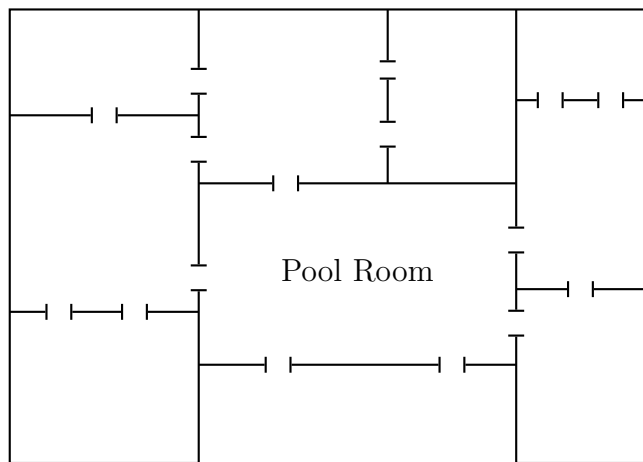


Figure 5: The floor of Von Diamond Estate