Review for the midterm for Math 31

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1. Calculate the following limits. If a limit does not exist state so and explain why.

   A. $\lim\limits_{x\to 0}\cos(x+\sin x)$ B. $\lim\limits_{x\to 0}\dfrac{x^2-9}{x^2+2x-3}$ C. $\lim\limits_{x\to 1^+}\dfrac{x^2+x-6}{x^2+2x-3}$ D. $\lim\limits_{x\to -4}\dfrac{x+4}{|x+4|}$

   E. $\lim\limits_{z\to 3}\dfrac{z+2}{z^2-z-6}$ F. $\lim\limits_{w\to -2}\dfrac{w+1}{4+8w+5w^2+w^3}$ G. $\lim\limits_{t\to 2}\dfrac{2t^2+7t+6}{t^4-16}$ H. $\lim\limits_{z\to 2^-}\dfrac{|z-2|}{z-2}$

   I. $\lim\limits_{x\to 9}\dfrac{3-\sqrt{x}}{x-9}$ J. $\lim\limits_{u\to 2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x}$ K. $\lim\limits_{t\to 25}\dfrac{\sqrt{t}-5}{\sqrt{t-25}}$ L. $\lim\limits_{x\to \pi/2^+}\tan x$

   M. $\lim\limits_{x\to 0}\sin\dfrac{1}{x}$ N. $\lim\limits_{x\to 0}x^4\sin\dfrac{1}{x}$ O. $\lim\limits_{x\to 0}\dfrac{3x}{\sin 3x}$ P. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}$ Q. $\lim\limits_{x\to -3}\dfrac{\sin(x+3)}{x^2+x-6}$

   R. $\lim\limits_{x\to 0}\dfrac{\sin x}{\tan x}$ S. $\lim\limits_{x\to 0}\dfrac{8x-\tan 6x}{\sin 6x}$ T. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x-\tan 3x}$ U. $\lim\limits_{\phi\to 0}\dfrac{\sin(\cos(\phi))}{\sec\phi}$

2. Find the real number(s) a so that f is continuous at all real numbers:

$$f(x)=\begin{cases} x^2-2a & \text{if } x<-1\\ 3x+3a & \text{if } x\geq -1\end{cases}$$

3. Prove that the equation $x^4-5x^3+7x^2+8x-3=0$ has a solution in the interval $(0,1)$.

4. Consider the function $f(x)=\dfrac{1}{x-1}$; we have that $f(0)=-1$ and $f(2)=1$. However even though 0 is between -1 and 1 there is no c in $(-1,1)$ with $f(c)=0$. Does this contradict the Intermediate Value Theorem? Why or why not?

5. A Tibetan monk leaves the monastery at 7:00 A.M. and takes his usual path to the top of the mountain, arriving at 7:00 P.M. The following morning, he start at 7:00 A.M. at the top of the mountain and takes the same path back, arriving at the monastery at 7:00 P.M. Prove that there is at least one point of the path that the monk will cross at exactly the same time both days.

6. Give an example of a function that
   (a) has a jump discontinuity at $x=-5$.
   (b) has a removable singularity at $x=0$.
   (c) has an infinite discontinuity at $x=3$.
   (d) is continuous everywhere except at $x=0$ and the discontinuity is not jump, removable or infinite.

7. Calculate each of the following derivatives using the definition of the derivative as the limit of the difference quotients:

   A. $\dfrac{d}{dx}\left(x^3-3x^2+5x-2\right)$ B. $\left(\dfrac{1}{x^2}\right)'$ C. $\left(\dfrac{x+1}{x+2}\right)'$ D. $\dfrac{d}{dx}\left(\sqrt{2-3x}\right)$

8. Calculate y':

   A. $y = (x^3 + x)^5$ B. $y = \dfrac{-x^2 + x - 2}{\sqrt{x}}$ C. $y = \sqrt[3]{1 + \sec x}$ D. $y = \dfrac{1}{(x-4)^{42}}$

   E. $y = \cos(\sin 3x)$ F. $y = \dfrac{x^2 + 1}{\csc x}$ G. $y = \sin^3\left(\dfrac{x-1}{x^2+1}\right)$ H. $y = \pi \sin x \cos x \cot x$

   I. $y = \tan\sqrt{1-x}$ J. $x^4y - 3xy^4 + x^2y^2 = 3x + 4y$ K. $\sin xy = x^2 - y$

   L. $y = \dfrac{\sin x}{x}$ M. $y = \dfrac{(x+2)^2}{x^2+16}$ N. $y = \sqrt{x}\sin\sqrt{x}$ O. $y = \dfrac{1}{\sqrt[3]{1+\sqrt{x}}}$

9. Find an equation of the tangent to the curve at the given point:
   (a) $y = 4\sin^2 x$, at the point $(\frac{\pi}{6}, 1)$
   (b) $y = \dfrac{x^2 - 4}{x^2 + 4}$, at the point $(0, -1)$
   (c) $y = \sqrt{4 - 2\sin x}$, at the point $(0, 2)$
   (d) $x^3 + 3x^2y - 2xy^2 - y^3 = 49$, at the point $(3, 2)$
   (e) $x^{2/3} + y^{2/3} = 4$, at the point $(-3\sqrt{3}, 1)$

10. At what points on the curve $y = \sin x - \cos x$, $0 \le x \le 2\pi$ is the tangent line horizontal?

11. Find the points on the ellipse $2x^2 + y^2 = 1$ where the tangent line has slope 1.

12. Find a parabola $y = ax^2 + bx + c$ that passes through the point $(1, 4)$ and whose tangent lines at $x = 1$ and $x = 5$ have slopes 6 and 2, respectively.

13. How many tangent lines to the curve $y = \dfrac{x}{1+x}$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?

14. Give an example of a function f whose graph has a tangent line at $x = 0$ but $f'(0)$ does not exist.

15. A particle moves on a vertical line according to the law of motion

$$s(t) = 2t^3 - 9t^2 + 12t + 3, \qquad t \ge 0$$

    where t is measured in seconds and s in meters.
    (a) Find the velocity and the acceleration of the particle at time t.
    (b) When is the particle moving upward and when is it moving downward?
    (c) When is the particle speeding up and when is it slowing down?
    (d) Find the total distance traveled by the particle during the first six seconds.

16. A ladder 5 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3 m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 4 m from the wall?

17. Boat A travels west at a 50 miles per hour and boat B travels north at 60 miles per hour. The two boats are going to collide in 3 hours. At what rate are the two boats approaching each other 1 hour before the collision?

18. Use the linearization of $f(x) = \sqrt{25 - x^2}$ near 3 to approximate $f(3.1)$.

19. Use appropriate linear approximations to estimate the following:

    A. $\sqrt{9.04}$ B. $\sin 0.02$ C. $(1.03)^{-1/3}$ D. $\sqrt[3]{0.97}$