

Sixth Quiz for Math 31

Name: KEY

1. (25 points) For a third degree polynomial $p(x)$ we have that $p(0) = -2$, $p'(0) = 3$, $p''(0) = 5$, and $p'''(x) = 12$. Find $p(x)$.

Let $p(x) = ax^3 + bx^2 + cx + d$. Then $p(0) = d \Rightarrow d = -2$

Also $p'(x) = 3ax^2 + 2bx + c$ So $p'(0) = c \Rightarrow c = 3$

$p''(x) = 6ax + 2b$ So $p''(0) = 2b \Rightarrow 5 = 2b \Rightarrow b = \frac{5}{2}$

$p'''(x) = 6a$ So $p'''(0) = 6a \Rightarrow 12 = 6a \Rightarrow a = 2$

Thus $p(x) = 2x^3 + \frac{5}{2}x^2 + 3x - 2$

2. (25 points) Find $\int x(x-2)^8 dx$

Let $u = x - 2$ then $du = dx$

and $x = u + 2$.

$$\int x(x-2)^8 dx = \int (u+2)u^8 du = \int (u^9 + 2u^8) du = \frac{u^{10}}{10} + \frac{2u^9}{9} + C$$

$$= \frac{(x-2)^{10}}{10} + \frac{2(x-2)^9}{9} + C$$

3. A particle moves along a straight line with velocity $v(t) = 3t - 2$.

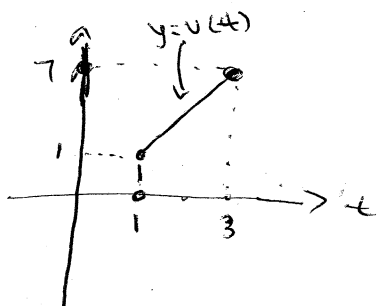
(a) (25 points) Find the displacement of the particle for the time interval $1 \leq t \leq 3$.

The displacement is $\int_1^3 v(t) dt = \int_1^3 (3t - 2) dt$

$$= \left[\frac{3t^2}{2} - 2t \right]_1^3$$
$$= \frac{3 \cdot 9}{2} - 2 \cdot 3 - \left(\frac{3}{2} - 2 \right)$$
$$= \frac{27 - 12 - 3 + 4}{2}$$
$$= \frac{16}{2}$$
$$= 8$$

(b) (25 points) Find the total distance covered by the particle in the time interval $1 \leq t \leq 3$.

The distance is $\int_1^3 |v(t)| dt$.



Now for $1 \leq t \leq 3$, $v(t) \geq 0$

$$\text{So } \int_1^3 |v(t)| dt = \int_1^3 v(t) dt = 8.$$