Quiz 2
Take home

You should fully justify your answers. Do all your work on separate paper, and make sure to *print* your name in the first sheet and staple all the sheets together. **Unstapled, loose pieces of paper will not be graded.** This quiz is due on Monday, September 19, at 8:00am.

1. Does the limit $\lim_{x\to\infty} \sin x$ exist?

2. Find the limit
$$\lim_{x\to\infty} \frac{\sin x}{x}$$
Hint. Use the Squeeze theorem.

3. Explain *in detail* why the function
$$f(x) = \frac{(2+x)^3 - 7}{1+x^2} - \sqrt{x^2+3} - \sin(\cos(3x))$$
is continuous on $(-\infty, \infty)$.

4. Find the points that each of the following functions is discontinuous and identify the nature of the discontinuity:
   (a) $f(x) = \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
   (b) $g(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$
   (c) $g(x) = \begin{cases} \dfrac{3}{(x-5)^2} & \text{if } x \neq 5 \\ 5 & \text{if } x = 5 \end{cases}$

5. Find the real number a so that the function defined by
$$f(x) = \begin{cases} 2x - a & \text{if } -\infty \leq x \leq \pi \\ \sin x & \text{if } \pi < x < \infty \end{cases}$$
is continuous on $\mathbb{R}$.

6. Give an example of a function that
   (a) has a jump discontinuity at $x = -5$.
   (b) has a removable singularity at $x = 0$.
   (c) has an infinite discontinuity at $x = 3$.
   (d) is continuous everywhere except at $x = 0$ and the discontinuity is not jump, removable or infinite.

7. Prove that the equation $2^x = x^2$ has a solution in the interval $[-1, 0]$.[1] Use a computer or a calculator to approximate that solution to the second decimal place.

[1] This equation has also two obvious positive solutions can you find them?

8. **Extra Credit** Use the fact that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

to evaluate the following limits

(a) $\lim_{x \to 0} \frac{\sin 5x}{5x}$

(b) $\lim_{x \to 0} \frac{\sin 3x}{x}$

(c) $\lim_{x \to 0} \frac{\sin 2x}{3x}$

(d) $\lim_{x \to 0} \frac{\cos x - 1}{x}$

Hint. Remember Pythagoras!