

Second Review Test

1. Calculate the following limits. Your answer should be a real number, ∞ , $-\infty$, or *does not exist*. If you think that a limit does not exist, explain why.

A. $\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 6x}$ B. $\lim_{x \rightarrow 3} \frac{2x - 6}{x^2 + x - 12}$ C. $\lim_{x \rightarrow \infty} \frac{8x^3 - 12x^2 + 6x - 1}{4x^3 - 2x^2 + 6x - 3}$ D. $\lim_{x \rightarrow 2} \frac{\sqrt{7x + 2} - 4}{3x - 6}$

E. $\lim_{x \rightarrow 1^-} \frac{x + 5}{x^2 + 4x - 5}$ F. $\lim_{x \rightarrow -7} \frac{|x + 7|}{x + 3}$ G. $\lim_{x \rightarrow 0} x^4 \sin \frac{\pi}{x}$

2. Compute the following derivative using the definition

$$\frac{d}{dx} \left(2x + \frac{1}{x} \right)$$

3. Compute $f'(x)$ where

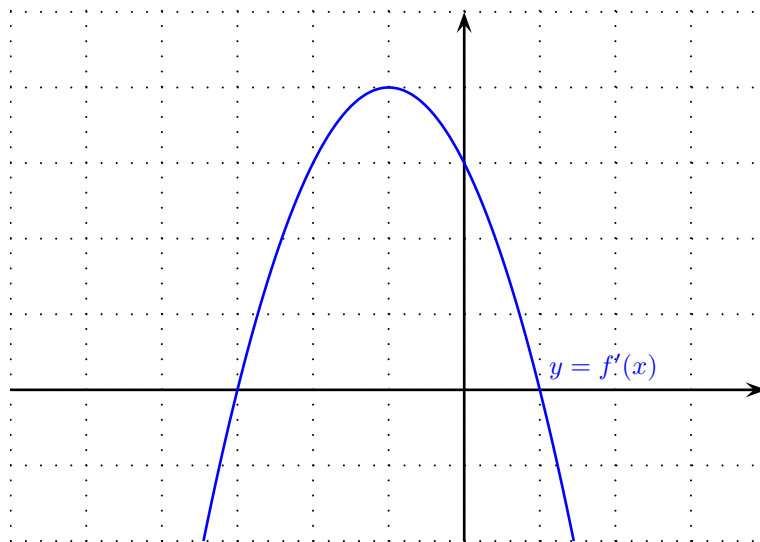
$$f(x) = x^2 \sin(\sqrt{x})$$

4. Prove that the equation

$$x^3 - 6x^2 + 15x + 6 = 0$$

has exactly one real solution.

5. The function f has the property that $f(0) = 1$, while the graph of its derivative is shown in the following picture:



Draw a rough but accurate graph of f .

6. Use Newton's method to estimate $\sqrt{5}$. Your estimate should be correct to four decimal digits.
7. Draw a qualitatively accurate graph of

$$f(x) = \frac{x^2 + 1}{x^2 - 1}$$

The graph should correctly indicate any x and y intercepts, possible asymptotes, extrema and inflection points, the intervals where the function is increasing or decreasing, and the intervals where the function is concave upward or downward.

8. Find the equation of the line tangent to the graph of

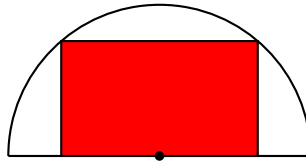
$$y = \frac{x^2 - 1}{x^2 + 1}$$

at the point $(0, -1)$.

9. Consider the function $f(x) = x^3 - x^2 - 4x + 11$.

- (a) Verify that f satisfies the hypotheses of Rolle's theorem on the interval $[-2, 2]$.
(b) Find all c that satisfy the conclusion of Rolle's theorem.

10. A rectangle is inscribed in a semicircle of radius 2 as shown below. What is the largest area among all such rectangles?



11. Estimate $\int_0^1 (x - x^3) dx$ with a Riemann sum using four subintervals and choosing the left endpoint in every subinterval as the representative point.
12. Calculate the following definite integral:

$$\int_{-5}^5 \frac{x^5 - 3x^3 + 2x}{3 + \cos x + \sin^2 x}$$

13. Find the indefinite integrals

- (a) $\int x^2 \sqrt{x^3 - 5} dx$
(b) $\int x^3 \sqrt{x^2 - 5} dx$

14. A particle moves in a line with velocity $v(t) = -3 \sin 2t$. For the time interval $[0, \pi]$, find:

- (a) the displacement of the particle,
(b) the total distance traveled by the particle.

15. Find the area of the region contained between, $y = x^2 - x$, $y = 0$, $x = -1$, and $x = 3$.