First Review Test

1. Calculate $\left(x^2 - 3x + 5\right)'$ using the definition of the derivative as a limit of difference quotients.

2. Consider the function $f(x) = \sin x$.
   (a) Verify that f satisfies the hypotheses of Rolle's theorem on the interval $[0, \pi]$.
   (b) Find all c that satisfy the conclusion of Rolle's theorem.

3. Let f be defined in a domain symmetric with respect to 0, i.e. for all x if x is in the domain so is $-x$.
   (a) Prove that if f is even then f' is odd.
   (b) Prove that if f is odd then f' is even.

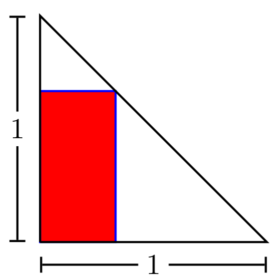
4. Let f be an odd function integrable over the interval $[-a, a]$. Prove that

$$\int_{-a}^{a} f(x)\, dx = 0$$

5. A number a is called a fixed point of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x then f has at most one fixed point.

6. Use Newton's method to approximate $\sqrt[3]{7}$ to five decimal places.

7. Consider a isosceles right triangle with legs of length 1 and a rectangle inscribed inside it in such a way that its sides are parallel to the legs of the triangle, as shown bellow. Find the dimensions of the rectangle so that its area is the maximum possible.



8. Find an equation for the line tangent to the graph of

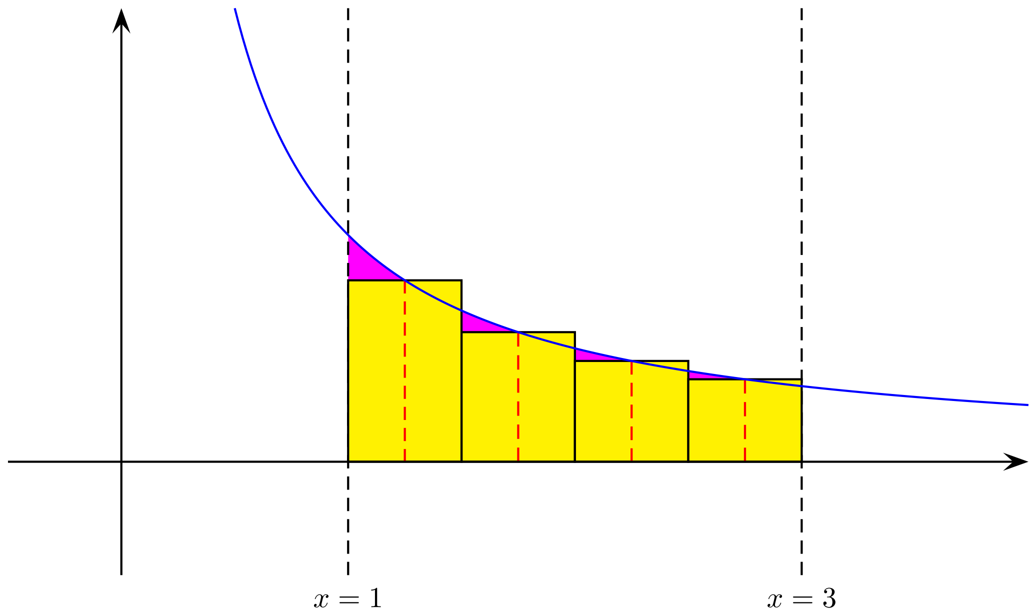
$$y \sin x^2 = x \sin y^2$$

at the point $(\sqrt{\pi}, \sqrt{\pi})$.

9. Calculate the following integrals:
   (a) $\displaystyle\int \left(x + \frac{1}{x}\right)^2 dx$
   (b) $\displaystyle\int x(3x-1)^{10}\, dx$
   (c) $\displaystyle\int \frac{\sin x}{\sqrt{1+\cos x}}\, dx$

(d) $\displaystyle\int_0^{\frac{\pi}{2}} \sin(\cos x)\sin x\,dx$

10. Consider the region bounded by the graph of $y = \dfrac{1}{x}$, the x–axis and the vertical lines $x = 1$ and $x = 3$.



(a) Partition the interval $[1, 3]$ into four equal subintervals and calculate the midpoint of each of these intervals.

(b) Calculate the area of each of the corresponding rectangles.

(c) Calculate the corresponding midpoint Riemann sum for the integral: $\displaystyle\int_1^3 \frac{dx}{x}$.

11. Assume that the acceleration due to gravity near the surface of the earth is $g = 10\,\mathrm{m/s^2}$ and that there are no forces due to friction, resistance of the air, etc. From the edge of a $50\,\mathrm{m}$ building, a ball is thrown upwards with initial velocity of $5\,\mathrm{m/s}$.

(a) Find the low of motion for the ball, i.e. an expression for $s = s(t)$.

(b) When will the ball hit the ground?

(c) What's the highest point that the ball will reach and when will it reach it?

(d) How much is the total distance that the ball will cover until it hits the ground?

12. Let $g(x) = \dfrac{4 + 6x}{\sqrt{x}}$

(a) Sketch a qualitatively accurate graph of f.

(b) Find the area contained between the graph of f, the x–axis, and the lines $x = \dfrac{1}{4}$ and $x = 4$

13. Let $f(x) = x^4 - 5x^2 + 4$.

(a) Sketch a qualitatively accurate graph of $y = f(x)$.

(b) Sketch a qualitatively accurate graph of $y = |f(x)|$.

(c) Find the area of the region bounded by the curves $y = f(x)$, $y = 0$, $x = -2$, and $x = 3$.

(d) Which one is larger $f(1.4567898765687654987)$ or $f(1.4567898765687654985)$?

14. A particle is moving on a line according to the law of motion $s(t) = t^4 - 5t^2 + 4$.

    (a) Find the displacement of the particle for the time interval $0 \leq t \leq 2$.

    (b) Find the total distance covered by the particle for the time interval $0 \leq t \leq 2$.

15. A particle is moving on a line with velocity given by $v(t) = t^4 - 5t^2 + 4$.

    (a) Find the displacement of the particle for the time interval $0 \leq t \leq 2$.

    (b) Find the total distance covered by the particle for the time interval $0 \leq t \leq 2$.

    (c) Find the displacement and the total distance covered for the time interval $0 \leq t \leq 1$.

    (d) If at $t = 0$ the particle was at $s(0) = -4$, find the law of motion for this particle (i.e. a formula for $s(t)$).

16. Find the area of the region determined by the graphs of $y = \cos x$, $y = 0$, $x = 0$, and $x = 4$.