BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 31   Exam 1
Nikos Apostolakis   October 21, 2011

Name: _____

Directions: Write your answers in the provided booklets. Make sure to indicate
which answer belongs to which question. To get full credit you *must* show all your
work. Simplify your answers whenever possible. Be certain to indicate your final
answer clearly. This exam has a total of 1000 points. The perfect score for this exam
is 1000 points.

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1. Find the following limits. Your answer should be a real number, $+\infty$, $-\infty$, or *Does Not Exist*.

   (a) (50 points) $\displaystyle\lim_{x \to -5} \frac{x^2 - 2x - 35}{x + 5}$

   (b) (100 points) $\displaystyle\lim_{x \to 0} \frac{\sin 5x}{3x - \tan 5x}$

   (c) (50 points) $\displaystyle\lim_{x \to -7} \frac{|x + 7|}{x + 7}$

   (d) (50 points) $\displaystyle\lim_{x \to 0} \frac{x^3 - 6x^2 + 8x}{x^5 - x^4 - 12x^3}$

2. (100 points) Prove that the equation $5x^3 - 7x^2 + 8x - 1 = 0$ has a solution in the interval $(0, 1)$.

3. (150 points) Calculate $\dfrac{d}{dx}\left(\sqrt{2x + 3}\right)$ using the definition of the derivative as a limit of the difference quotients.

4. Calculate the following derivatives. Simplify your answer as much as possible:

   (a) (50 points) $\left(\dfrac{(x - 3)^2}{x^2 - 9}\right)'$

   (b) (50 points) $\left(\sqrt{x^2 + 1}\sin\sqrt{x^2 + 1}\right)'$

5. (150 points) Find the equation of the line tangent to the curve

$$y^3 + x^3 = 2xy^2 + x - 1$$

   at the point $(-2, -1)$

6. A particle moves on a vertical line according to the law of motion

$$s(t) = t^3 - 6t^2 + 9t + 5, \qquad t \geq 0$$

   where t is measured in seconds and s in meters.

   (a) (50 points) When is the particle moving upward and when is it moving downward?

   (b) (50 points) When is the particle speeding up and when is it slowing down?

   (c) (50 points) Find the total distance traveled by the particle in the first four seconds.

7. (100 points) Use linear approximation to estimate $\sqrt[3]{7.97}$.