DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MATH 31   Exam 1, Take II

Nikos Apostolakis   October 27, 2011

Take Home
Due: Monday, October 31

Directions: Make sure to indicate which answer belongs to which question. To get full credit you *must* show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. The perfect score for this exam is 1000 points.

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1. Find the following limits. Your answer should be a real number, $+\infty$, $-\infty$, or *Does Not Exist*.

   (a) (50 points) $\displaystyle\lim_{x\to\frac{1}{2}} \frac{2x^2 + x - 1}{2x - 1}$

   (b) (50 points) $\displaystyle\lim_{x\to 3} \frac{\sqrt{4x - 3} - 3}{x - 3}$

   (c) (50 points) $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{\sin 3x}$

   (d) (50 points) $\displaystyle\lim_{x\to 0} \frac{x^3 + 2x^2 - 3x}{x^4 - 4x^3 + 3x^2}$

   (e) (50 points) $\displaystyle\lim_{x\to 5} \sqrt{x - 5}\,\sin\frac{\pi}{x - 5}$

2. (100 points) Prove that the equation $\cos x = x$ has a solution in the interval $(0, \frac{\pi}{2})$.

3. (150 points) Calculate $\left(\dfrac{x}{x + 1}\right)'$ using the definition of the derivative as a limit of the difference quotients.

4. Find y':

   (a) (50 points) $\cos\left(\sin\sqrt{x}\right)$

   (b) (50 points) $\dfrac{x^2 - 3x + 2}{\sqrt{x}}$

(c) (50 points) $xy = \tan(x + y) - x$

5. (100 points) Let $f(x) = x\sqrt{5 - x}$. Find the equation of the line tangent to the graph of $y = f(x)$ at the point $(1, 2)$.

6. A particle moves on a vertical line according to the law of motion

$$s(t) = t^3 - 3t^2 - 4, \qquad t \geq 0$$

where t is measured in seconds and s in meters.

(a) (50 points) When is the particle moving upward and when is it moving downward?

(b) (50 points) When is the particle speeding up and when is it slowing down?

(c) (50 points) Find the total distance traveled by the particle in the first three seconds.

7. (100 points) Use linear approximation to estimate $\sqrt[4]{80.75}$.