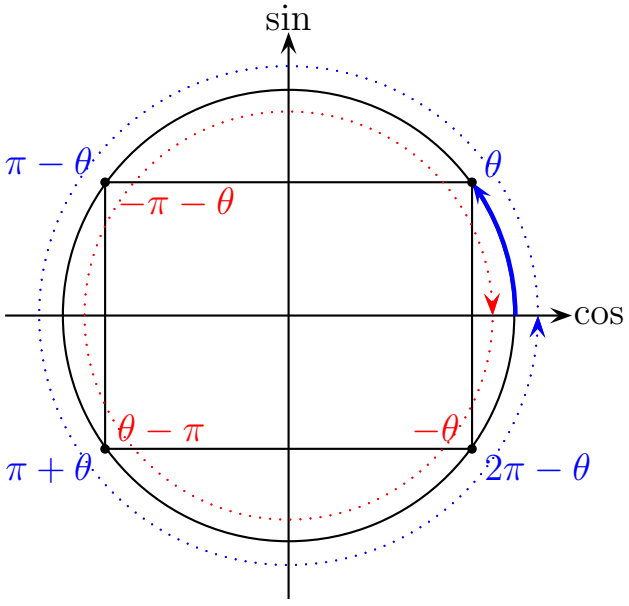
1 Some commonly occurring angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined

2 Arcs in the unit circle

If we start with an arc θ in the first quadrant, then in the counterclockwise direction the arc symmetric with respect to the sine axis is $\pi - \theta$, the arc symmetric to that with respect to the cosine axis is $\pi + \theta$, and finally the arc symmetric to that with respect to the sine axis is $2\pi - \theta$. Going in the counterclockwise direction the arc symmetric to θ with respect to the cosine axis is $-\theta$, the arc symmetric to that with respect to the sine axis is $\theta - \pi$, and the arc symmetric to that with respect to the cosine axis is $-\pi - \theta$.

Points in the circle that are in the same horizontal line have the same sine and opposite cosines; points that are on the same vertical line have the same cosine and opposite sines. Also antidiametrical points on the circle (i.e. θ and $\pi + \theta$) have the same tangent and cotangent but opposite sines and cosines.



$$\sin(\pi - \theta) = \sin(\theta) \qquad \sin(-\pi - \theta) = \sin(\theta)$$
$$\cos(\pi - \theta) = -\cos(\theta) \qquad \cos(-\pi - \theta) = -\cos(\theta)$$

$$\sin(\pi + \theta) = -\sin(\theta) \qquad \sin(\theta - \pi) = -\sin(\theta)$$
$$\cos(\pi + \theta) = -\cos(\theta) \qquad \cos(\theta - \pi) = -\cos(\theta)$$

$$\sin(2\pi - \theta) = -\sin(\theta) \qquad \sin(-\theta) = -\sin(\theta)$$
$$\cos(2\pi - \theta) = \cos(\theta) \qquad \cos(-\theta) = \cos(\theta)$$

3 Exercises

1. Calculate the following. Give exact answers whenever possible.
   (a) $\sin^{-1}\left(\frac{1}{2}\right)$
   (b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
   (c) $\cos\left(\sin^{-1}\left(\frac{3\sqrt{11}}{10}\right)\right)$
   (d) $\tan\left(\cos^{-1}\left(-0.4\right)\right)$
   (e) $\cos^{-1}\left(\cos\left(\frac{23\pi}{6}\right)\right)$
   (f) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$
   (g) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$
   (h) $\cos^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right)$
2. Give all solutions of the following equations in the interval $[0, 2\pi)$.
   (a) $\sin x = -\frac{\sqrt{3}}{2}$
   (b) $\cos x = \frac{\sqrt{2}}{2}$
   (c) $\sin x = \frac{1}{2}$
   (d) $\cos x = -\frac{1}{2}$
   (e) $\sin x = \frac{1}{3}$
   (f) $4\cos^2 x - 3 = 0$
   (g) $2\sin^2 x + \sin x = 0$
   (h) $2\sin^2 x + \sin x - 1 = 0$
   (i) $4\cos^3 x = \cos x$
   (j) $9\sin^4 x - 8\sin^2 x = 1$
3. Find all solutions of the equations of the previous exercise in the interval $(-\pi, \pi]$.