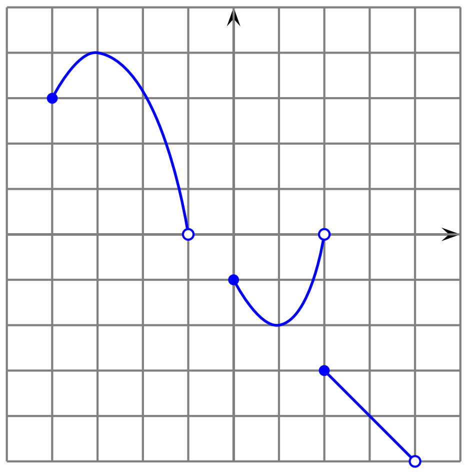
Second Quiz for Math 30

The answers

1. Find the domain and range of the function whose graph is shown bellow. Also determine the intervals at which the function is increasing or decreasing.



Answer. Domain is $[-4,-1) \cup [0,4)$. Range is $(-5,-3] \cup [-2,0) \cup (0,4]$. The function is increasing on $[-4,-3]$ and $[1,2)$; decreasing on $[-3,-1)$, $[0,1]$, and $[2,4)$. □

2. Find the domain of the following two functions:

(a) $f(x) = \sqrt{4-3x}$

Answer. We need $4-3x \geq 0$ or equivalently $\frac{4}{3} \geq x$. So the domain is $\left(-\infty, \frac{4}{3}\right]$. □

(b) $g(x) = \dfrac{3x}{x^2-16}$

Answer. We need $x^2 - 16 \neq 0$ or equivalently $x \neq \pm 4$. So, using interval notation, the domain is $(-\infty,-4) \cup (-4,4) \cup (4,\infty)$. □

(c) $f(x) = \log_{10}(3+x^2)$

Answer. We need $3 + x^2 > 0$. But this is true for all real numbers x. Therefore the domain is $\mathbb{R}$, the set of all real numbers □

3. Verify that the following is a pair of inverse functions: $f(x) = x^2 + 2x + 1$ with domain $[-1,\infty)$ and $g(x) = \sqrt{x} - 1$.

Answer. We'll show that

1. for all x in the domain of g, $f(g(x)) = x$, and
2. for all x in the domain of f, $g(f(x)) = x$.

We have:

$$\begin{aligned} f(g(x)) &= \left(\sqrt{x}-1\right)^2 + 2\left(\sqrt{x}-1\right) + 1 \\ &= x - 2\sqrt{x} + 1 + 2\sqrt{x} - 2 + 1 \\ &= x \end{aligned}$$

while,

$$\begin{aligned} g(f(x)) &= \sqrt{x^2+2x+1}-1 \\ &= \sqrt{(x+1)^2}-1 \\ &= |x+1|-1 \\ &= x+1-1 \\ &= x \end{aligned}$$

where to go from the third to the fourth line we used the fact that the domain of f is $[-1, \infty)$ and so for x in the domain of f, $|x+1| = x+1$. □

4. Find the domain, the range and the formula for f^{-1}, where

$$f(x) = \frac{x}{3-x}$$

Answer. As a relation f is given by the equation:

$$y = \frac{x}{3-x}$$

So the f^{-1} as a relation is

$$x = \frac{y}{3-y}$$

To find the formula for $f^{-1}(x)$ we solve for y:

$$\begin{aligned} x = \frac{y}{3-y} &\iff x(3-y) = 3 \\ &\iff 3x - xy = y \\ &\iff 3x = y + xy \\ &\iff 3x = y(1+x) \\ &\iff \frac{3x}{1+x} = y \end{aligned}$$

Therefore the formula for $f^{-1}(x)$ is

$$f^{-1}(x) = \frac{3x}{1+x}$$

From the formula we see that the domain of f^{-1} is $(-\infty, -1) \cup (-1, \infty)$.

The range of f^{-1} is the domain of f so from the formula of f we see that the range of f^{-1} is $(-\infty, 3) \cup (3, \infty)$. □