

Homework on Polynomial Functions I

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Please note: You should fully justify your answers.

1. Sketch a rough graph for each of the following polynomial functions. The graph should correctly reflect the end behavior, the behavior near x -intercepts and the number of turning points. The y -intercept should also be correctly marked.

(a) $f(x) = x(x + 3)(x + 2)(x - 1)(x - 2)$

(b) $g(x) = (x - 1)^2(x + 2)(x - 3)$

(c) $h(x) = (2x + 3)(x - 1)(3x - 2)$

(d) $f(x) = (2x + 1)^3(x - 2)^4$

(e) $g(x) = (x + 1)(x - 1)^2(x - 2)^2$

2. For each of graphs in Figure 1, give a polynomial function whose graph has that qualitative behavior. Your answer should be in expanded and simplified form, for example, don't give your answer as $f(x) = (x - 1)(x + 1)$ but as $f(x) = x^2 - 1$ instead.

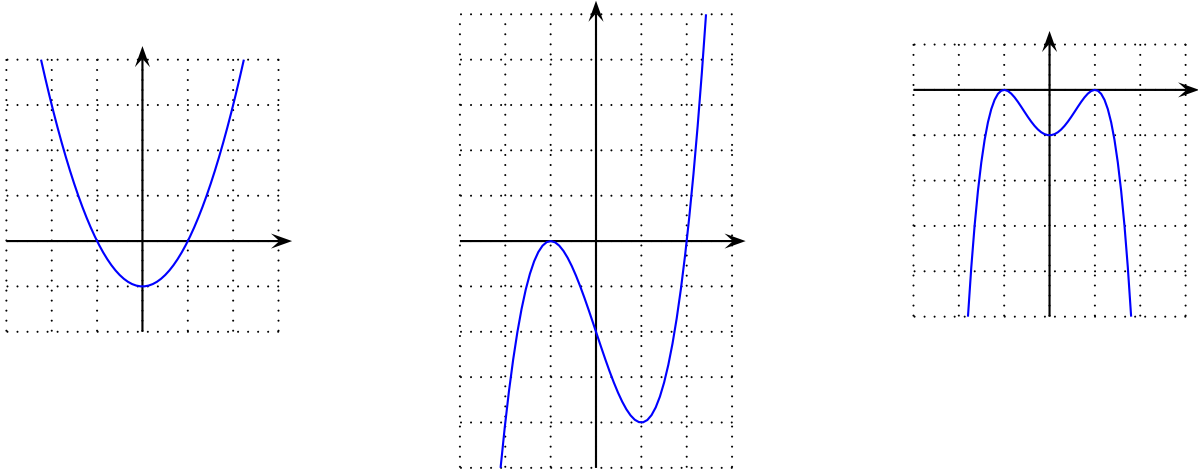


Figure 1: The graphs of Question 2

3. Solve the following inequalities. Give your answer using interval notation.

(a) $(x - 1)(x + 3)(x - 4) \geq 0$

(b) $(x + 2)^2(3 - x)(x - 1)^3 > 0$

(c) $(x - 7)(2x + 4)^4(x + 3)(x - 4) \leq 0$

(d) $(x - 4)(x + 3)^2(x - 2)^3(x + 1)^4 < 0$

4. Use SAGE to solve the following inequalities by graphing appropriate functions:

(a) $x^3 - 6x^2 + 12x - 8 \geq 0$

(b) $x^4 - x^3 - 5x^2 - x - 6 < 0$

(c) $\frac{x - 3}{x + 4} > 0$

(d) $\frac{x^2 - 4}{x^3 - 27} \leq 0$

5. Given that $x = 5$ is a solution to the following equation

$$x^3 - 7x^2 + 15x - 25 = 0$$

find all solutions.

6. Given that $x = -2$ is a solution to the following equation:

$$x^4 - 5x^3 + x^2 + 5x - 50 = 0$$

solve the equation completely.

7. $x = 3$ is a solution to the equation

$$x^3 - 9x^2 + 27x - 27 = 0$$

Solve the equation completely.

8. Given that $x^2 + 1$ is a factor of the polynomial $x^4 + x^3 - 2x^2 + 4x - 24$ solve the following equation:

$$x^4 + x^3 - 2x^2 + 4x - 24 = 0$$

9. One of the numbers 1, -2, 3, 4 is a solution to the equation

$$x^3 - 3x^2 - 10x + 24 = 0$$

Solve the equation.

10. Find a cubic polynomial with zeros at $x = -1$, $x = 3$ and $x = 2$.

11. Find a fourth degree polynomial with real coefficients and zeros at $x = 3i$, $x = 2$, and $x = 0$.