

## Homework on Exponential and logarithmic functions

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**Please note:** You should fully justify your answers.

1. Simplify the following expressions. All variables are assumed positive:

(a)  $\log_3 27$

(b)  $\log_2 \frac{1}{32}$

(c)  $\log_2 2^{52} - 2^{\log_2 14} + \log_2 16$

(d)  $\log 0.001$

(e)  $\log_b \frac{1}{b^{42}}$

(f)  $e^{\ln(5+\sqrt{6x})}$

(g)  $\ln e^{7x^3}$

(h)  $\log_{42} 1$

2. For each of the following pairs of functions

1. Verify that it is a pair of inverse functions.

2. Find the domain and the range of each function in the pair.

3. Graph the two functions using SAGE.

(a)  $f(x) = \log(x - 3)$  and  $g(x) = 10^x + 3$

(b)  $f(x) = \log_2(2x + 5) - 3$  and  $g(x) = \frac{2^{x+3} - 5}{2}$

(c)  $f(x) = \ln \frac{2x + 5}{3x - 2}$  and  $g(x) = \frac{2e^x + 5}{3e^x - 2}$

(d)  $f(x) = 3^{3^x}$  and  $g(x) = \log_3(\log_3 x)$

3. Find the domain of the following functions:

(a)  $f(x) = \ln(7x + 2)$

(b)  $f(x) = \log_3(3 - 5x)$

(c)  $g(x) = \ln(2x^2 + 5x - 3)$

(d)  $g(x) = \log(x^2 - 6x + 9)$

(e)  $h(x) = \log_5(x^2 - 2x + 5)$

(f)  $f(x) = \ln(x^3 - 2x^2 - 5x + 6)$

(g)  $h(x) = \log_{11}(x^5 + 5x^4 - 5x^3 - 45x^2 + 108)$

(h)  $f(x) = \log_{42} \left( \frac{x - 2}{x + 1} \right)$

(i)  $f(x) = \log_{42}(x - 2) - \log_{42}(x + 1)$

4. Expand:

(a)  $\ln e^2 x^3 y^4 \sqrt{z}$

(b)  $\log \frac{10x^2 y^3}{z^4}$

(c)  $\log_2 \sqrt{\frac{x^3 y^5}{8z}}$

5. Contract:

(a)  $\ln 2 + \ln 3 - 2 \ln 6$

(b)  $\log_4 x^2 - \log_4 \sqrt[3]{x^5} + \log_4 x$

(c)  $1 + \log(x - 2) + \log(x + 2)$

6. Solve the following equations:

(a)  $5^x = \frac{1}{125}$

(b)  $3^x = \sqrt[4]{27}$

(c)  $4e^{-4x+1} = 8$

(d)  $4^x = 3^{x-1}$

(e)  $4^{2x-1} = e^{3x+2}$

(f)  $\log_2(x - 1) = 4$

(g)  $\log_3 \sqrt{x + 2} = 2$

(h)  $\log_2(x - 1) + \log_2(x + 3) = 5$

(i)  $\log_4(x + 2) + \log_4(x - 2) = \frac{5}{2}$

(j)  $\log_3 x + \log_3(x^2 + 11x + 15) = 3$

(k)  $\log x + 2 \log(x + 1) = 2$

(l)  $\ln x - \ln(x + 1) = 1$

(m)  $\log_2(2 - x) + \log_2(x + 2) - \log_2(1 - x) - \log_2(x + 1) = 2$

(n)  $\log 10^{2x} - 9 \cdot 10^x - 10 = 0$

(o)  $4^{2x} - 6 \cdot 4^x + 8 = 0$

(p)  $e^{2x} - 4e^x + 3 = 0$

(q)  $2^{3x} - 7 \cdot 2^{2x} + 14 \cdot 2^x - 8 = 0$

7. Find the inverse function for each of the following functions. Your answer should include the formula, the domain and the range.

(a)  $f(x) = 5^{x+3}$

(b)  $f(x) = 2 \cdot 3^{5x-1} - 4$

(c)  $g(x) = \ln(3x - 2)$

(d)  $h(x) = 3 \log_2(x + 1) - 4$

(e)  $g(x) = \ln(2x - 1) - \ln(x + 2)$