Ninth Set of Homework for Math 05

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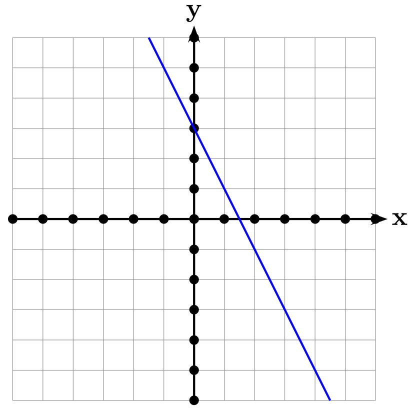
Please note: You should fully justify your answers.

1 Finding equations of lines

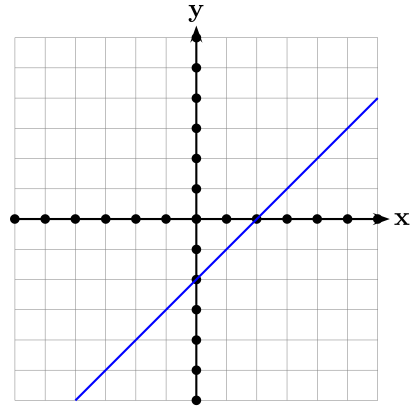
1. Find an equation of the line that:
   (a) has slope -2 and y–intercept 11. $y = -2x + 11$
   (b) has slope $-\frac{5}{2}$ and y–intercept 0. $y = -\frac{5}{2}x$
   (c) has slope $\frac{3}{4}$ and passes through the point $(0, -4)$ $y = \frac{3}{4}x - 4$
   (d) has the same slope as $2y - 4x = 10$ and the same y–intercept as $y = 5x - 3$. $y = 2x - 3$
   (e) has slope -5 and passes through the point $(-2, 3)$. $y = -5x + 7$
   (f) has slope 0 and passes through the point $(3, 5)$. $y = 5$
   (g) is vertical and passes through the point $(-3, 0)$. $x = -3$
   (h) passes through the points $(-5, 13)$ and $(1, -5)$. $y = -3x - 2$
   (i) passes through the points $(-2, 4)$ and $(1, 7)$. $y = x + 6$
   (j) passes through the points $(3, 0)$ and $(6, 2)$. $y = \frac{2}{3}x - 2$
   (k) passes through the points $(-1, 5)$ and $(-1, -3)$. $x = -1$
   (l) passes through $(0, 0)$ and $(3, -5)$. $y = -\frac{5}{3}x$
   (m) passes through the points $(2, 4)$ and $(-3, 4)$. $y = 4$
   (n) passes through the points $(0, 4)$ and $(-5, 0)$. $y = \frac{4}{5}x + 4$
   (o) passes through the points $\left(\frac{2}{3}, -\frac{1}{9}\right)$, and $\left(-\frac{15}{2}, -\frac{6}{5}\right)$. $y = \frac{2}{15}x - \frac{1}{5}$
   (p) has the same slope as $3x - 5y = -2$ and the same x–intercept as $-2x - 3y = 6$. $y = \frac{3}{5}x - 3$
   (q) has the same x–intercept as $-2x+3y = -2$ and the same y–intercept as $x-y = 3$. $y = -3x-3$
2. Find the equations for each of the lines in Figure 1.

2 Parallel lines, Perpendicular lines

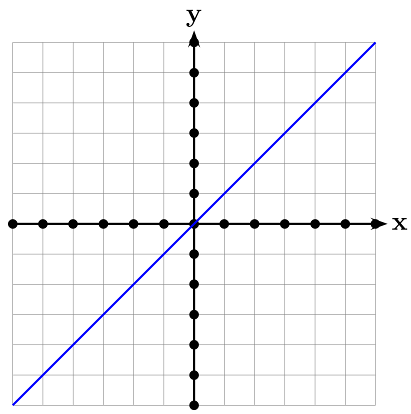
1. For each of the following pairs of lines, decide whether they are parallel, perpendicular or neither.
   (a) $y = 3x - 4$, $y = -3x + 2$ Neither
   (b) $y = \frac{2}{3}x$, $y = -\frac{3}{2}x + 9$ Perpendicular
   (c) $2x - 3y = 7$, $2x - 3y = 5$ Parallel
   (d) $3x + y = -2$, $-2x + 3y = 0$ Neither
   (e) $-5x + 2y = 8$, $2x + 5y = -3$ Perpendicular
   (f) $y = 3x + 8$, $3x + y = -3$ Neither



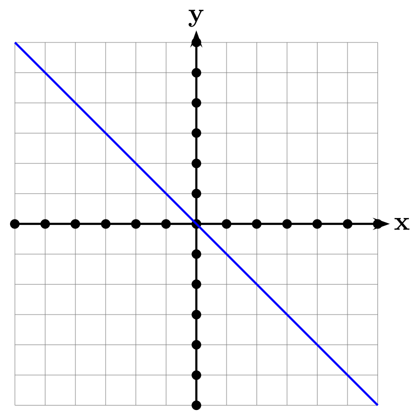
(a) $y = -2x + 3$



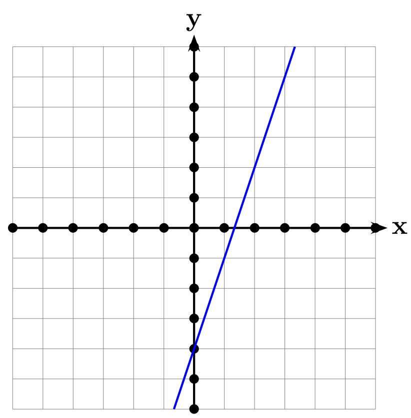
(b) $y = x - 2$



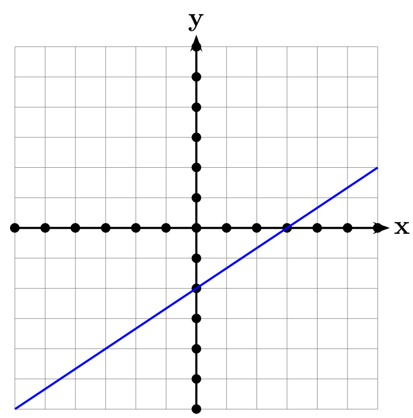
(c) $y = x$



(d) $y = -x$



(e) $y = 3x - 4$



(f) $y = \frac{2}{3}x - 2$

Figure 1: The lines of Question 2

(g) $y = 2x - 7$, $y = 2x + 9$ Parallel

(h) $y = 5x - 7$, $y = -\dfrac{x}{5} + 9$ Perpendicular

(i) $2x + 3y - 9 = 0$, $y = -\dfrac{2x}{3} - 2$ Parallel

2. Find an equation for the line that:

   (a) passes through $(-1, 3)$ and is parallel to the line $y = 3x - 5$. $y = 3x + 6$

   (b) is parallel to to $2x - 5y = 6$ and passes through $(1, -2)$. $2x - 5y = 12$

   (c) is parallel to $x = -3$ and passes through $(5, 9)$. $x = 5$

   (d) is perpendicular to $x = 2$ and passes through $(3, 4)$. $y = 4$

   (e) is perpendicular to $y = -\dfrac{x+2}{3}$ and passes through $(0, -2)$.

   (f) passes through the point $(3, 2)$ and is perpendicular to $2x - 3y = 5$. $3x + 2y = 13$

   (g) has the same y–intercept as $3x - 4y = 8$ and is parallel to $y = -5x + 11$. y = -5x -2

3. Verify that the following four points are the corners of a parallelogram.

$$P(-4, -9),\ Q(-2, -3),\ R(-4, -7),\ S(-6, -13)$$

   Answer. RQ and PS each have slope 2, so they are parallel. Also PQ and RS are parallel because each have slope 3. So, $PQRS$ is a parallelogram. □

4. Verify that the following three points are the corners of a right triangle.

$$A(2, 4),\ B(0, 0),\ C(4, 3)$$

   Answer. The slope of AB is $m_1 = 2$ and the slope of AC is $m_2 = -\dfrac{1}{2}$. Since $m_1 m_2 = -1$ it follows that AB and AC are perpendicular, so the angle A of ABC is a right angle. Therefore ABC is a right triangle. □

5. Verify that the following four points are the corners of a rectangle.

$$A(1, 1),\ B(4, 4),\ C(-1, 3),\ D(2, 6)$$

   Answer. By computing the slopes we see that AC and BD are parallel and so are AB and CD. Additionally, AC is perpendicular to AB. So $ABCD$ is a parallelogram with a right angle. So it has to be a rectangle. □

6. Consider again a line l with equation in standard form

$$Ax + By + C = 0$$

   where A, B, C are real numbers and at least one of A, B is non-zero.

   (a) Prove that a line with equation

$$Ax + By + D = 0$$

   where D is any number, is parallel to l.

   (b) Prove that a line with equation

$$Bx - Ay + D = 0$$

   where D is any number, is perpendicular to l.