

## Sixth Set of Homework for Math 05

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**Please note:** You should fully justify your answers.

### 1 Formulas

1. Solve each of the following equations for the stated variable. If you need to divide by a variable you should explicitly state that it is non-zero.

(a)  $F = ma$ , for  $a$ .  $a = \frac{F}{m}$ ;  $m \neq 0$

(b)  $ax + b = 0$ , for  $x$ .  $x = -\frac{b}{a}$ ;  $a \neq 0$

(c)  $2x - 3y = 6$ , for  $y$ .  $y = \frac{2}{3}x - 2$

(d)  $y = 3x - 5$ , for  $x$ .  $x = \frac{y + 5}{3}$

(e)  $Ax + By + C = 0$ , for  $y$ .  $y = -\frac{Ax + C}{B}$ ;  $B \neq 0$

(f)  $y = mx + b$ , for  $m$ .  $m = \frac{y - b}{x}$ ;  $x \neq 0$

(g)  $s = \frac{1}{2}gt^2$ , for  $g$ .  $g = \frac{2s}{t^2}$ ;  $t \neq 0$

(h)  $C = \frac{5}{9}(F - 32)$ , for  $F$ .  $F = \frac{9}{5}C + 32$

(i)  $2y = 3ax - 2x + 3$ , for  $x$ .  $x = \frac{3 - 2y}{2 - 3a}$ ;  $a \neq \frac{2}{3}$

(j)  $2a = \frac{3ax - b}{2b} - c$ , for  $a$ .  $a = -\frac{b + 2bc}{2b - 3x}$ ;  $2b - 3x \neq 0$

2. Find the numbers described in parts a, d of Exercise 2 of the Fourth set of homework.

a) The consecutive numbers are **4, 5**.

d) The number is **-13**.

3. The width of a rectangle is three less than twice its length.

(a) If the length of the rectangle is 7 inches how much is its width? **11 inches.**

(b) If the width of the rectangle is 21 inches how much is its length? **12 inches.**

(c) Find a formula that gives the length of the rectangle in terms of its width.  $l = \frac{w + 3}{2}$

(d) Write a formula for the perimeter  $P$  of this rectangle that involves only its length  $l$ .  $P = 6l - 6$

(e) If the perimeter of the rectangle is 24 inches find its dimensions (i.e. its length and its width).  
**Length is 5 inches and width is 7 inches.**

4. The temperature  $C$  in degrees Celsius and the temperature  $F$  in degrees Fahrenheit are related by the formula:

$$C = \frac{5(F - 32)}{9}$$

One day the numerical value of both temperature measurements was the same. What was the temperature that day? **-40°.**

## 2 Interval notation

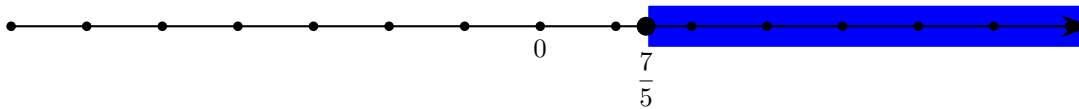
1. Write each of the sets described by the following mathematical sentences using interval notation, then graph the set in the real line.

- (a)  $-3 < x < 5$       $(-3, 5)$
- (b)  $-3 \leq x < 5$       $[-3, 5)$
- (c)  $-3 < x \leq 5$       $(-3, 5]$
- (d)  $-3 \leq x \leq 5$       $[-3, 5]$
- (e)  $7 > x > 3$       $(3, 7)$
- (f)  $-2 \geq x > -5$       $(-5, -2]$
- (g)  $x \geq 5$       $[5, \infty)$
- (h)  $x > -2$       $(-2, \infty)$
- (i)  $x < -3$       $(-\infty, -3)$
- (j)  $x \leq -3$       $(-\infty, -3]$

2. Write each of the sets described by the following English sentences using interval notation, then graph the set in the real line.

- (a) All real numbers that are greater or equal to seven fifths.

*Answer.*  $[\frac{7}{5}, \infty)$



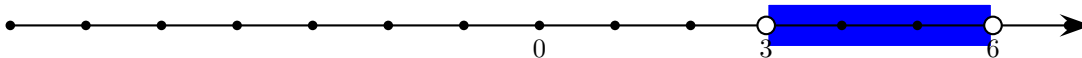
- (b) All real numbers that are less than zero.

*Answer.*  $(-\infty, 0)$



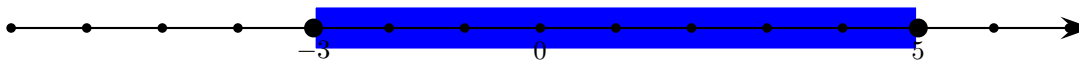
- (c) The set of real numbers that are larger than three and at the same time smaller than six.

*Answer.*  $(3, 6)$



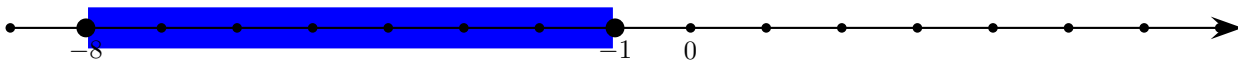
- (d) The set of numbers that are at most 5 and at least  $-3$ .

*Answer.*  $[-3, 5]$



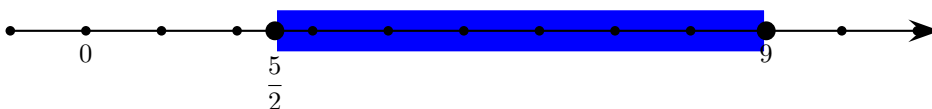
(e) Those numbers that are at least  $-8$  but no more than  $-1$ .

Answer.  $[-8, -1]$



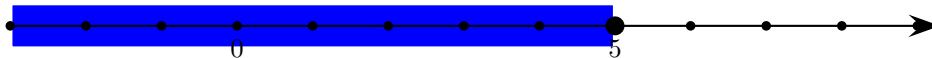
(f) Those numbers that are no less than  $\frac{5}{2}$  and no more than 9.

Answer.  $[\frac{5}{2}, 9]$



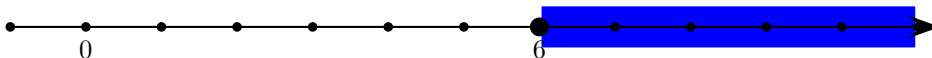
(g) Those numbers that are five or less.

Answer.  $(-\infty, -5]$



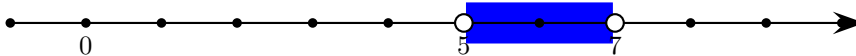
(h) Those numbers that are six or more.

Answer.  $[6, \infty]$



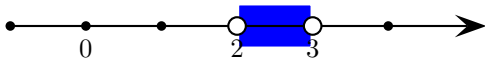
(i) The numbers strictly between five and seven.

Answer.  $(5, 7)$



(j) The numbers strictly between three and two.

Answer.  $(2, 3)$



(k) The numbers between negative two and five, inclusive.

Answer.  $[-2, 5]$



□

3. What intervals are described in the graphs of Figure 1

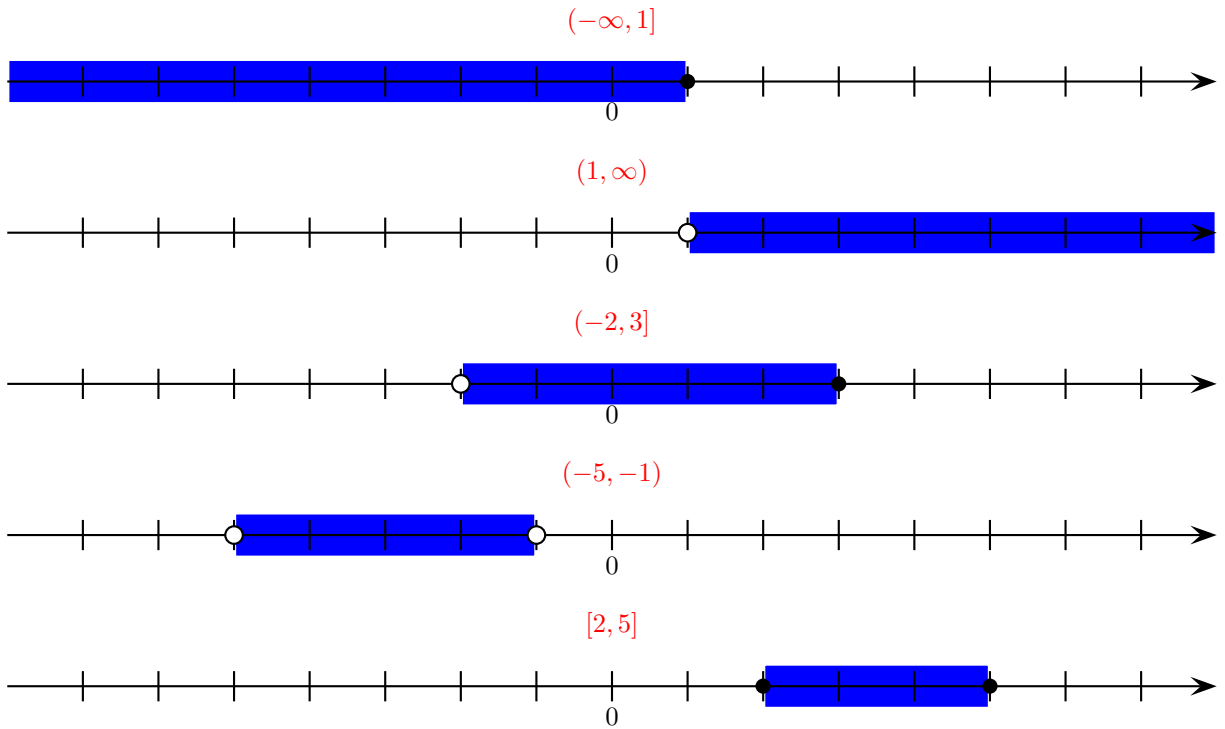


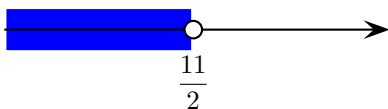
Figure 1: The graphs of Question 3

### 3 Solving Linear Inequalities

1. Solve each of the following inequalities. Give your answer in interval notation and sketch a graph of the solution.

(a)  $2x - 4 \leq 11$

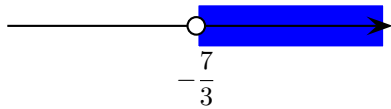
Answer.  $(-\infty, \frac{11}{2})$ . Graph is:



□

(b)  $-3x + 2 > 9$

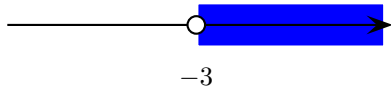
Answer.  $(-\frac{7}{3}, \infty)$ . Graph is:



□

(c)  $5x - 2 < 3x - 8$

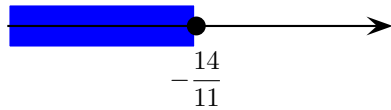
Answer.  $(-3, \infty)$ . Graph is:



□

(d)  $\frac{3}{2}x - 5 \geq 7x + 2$

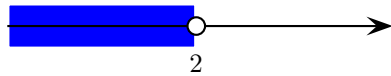
Answer.  $(-\infty, -\frac{14}{11}]$ . Graph is:



□

(e)  $-3(5x - 1) + 7 > -10x$

Answer.  $(-\infty, 2)$ . Graph is:

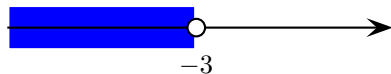


□

(f)  $2(x - 4) + 3x - 4 \leq 5x - 8$  **No solution**

(g)  $-5(x + 3) + 3x - 1 \geq 4(3x + 6) - 2x - 4$

Answer.  $(-\infty, -3)$ . Graph is:



□

(h)  $3(2x - 2) + 5 > 2(3x + 5) - 11$  **No solution**

2. Solve the inequalities in parts e, f of Exercise 2 of the Fourth set of homework.

Answer. e)  $13 < x$

f)  $x < -\frac{120}{11}$

□