Fifteenth Set of Homework for Math 05

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Please note: You should fully justify your answers.

1 Solving higher degree equations

1. Solve the following equations:

    (a) $5(3x - 7) = 0$ $\quad x = \dfrac{7}{3}$

    (b) $3x(x - 1) = 0$ $\quad x = 0$ or $x = 1$

    (c) $(x - 1)(x + 3)(2x + 5) = 0$ $\quad x = -3$ or $x = 1$ or $x = -\dfrac{5}{2}$

    (d) $2x(3x - 1)(x^2 + 1) = 0$ $\quad x = 0$ or $x = \dfrac{1}{3}$

    (e) $(x + 7)^2(x - 2)(x + 1) = 0$ $\quad x = -7$ or $x = 2$ or $x = -1$

2. Solve the following equations:

    (a) $x^2 - 7x = 0$ $\quad x = 0, \quad x = 7$

    (b) $x^2 - 64 = 0$ $\quad x = -8, \quad x = 8$

    (c) $3x^3 - 75x = 0$ $\quad x = 0, \quad x = -5, \quad x = 5$

    (d) $x^2 - x - 6 = 0$ $\quad x = -2, \quad x = 3$

    (e) $x^2 - 12x + 35 = 0$ $\quad x = 5, \quad x = 7$

    (f) $x^2 + 16x + 55 = 0$ $\quad x = -5, \quad x = -11$

    (g) $6x^2 - 5x + 1 = 0$ $\quad x = \dfrac{1}{3}, x = \dfrac{1}{2}$

    (h) $x^2 - 2x - 80 = 0$ $\quad x = -8, \quad x = 10$

    (i) $10x^3 - 29x^2 + 10x = 0$ $\quad x = \dfrac{2}{5}, \quad x = \dfrac{5}{2}$

    (j) $3x^2 + 12 = 0$ No solution

    (k) $2x^2 + x - 15 = 0$ $\quad x = -3, \quad x = -\dfrac{5}{2}$

    (l) $18x^2 + 29x + 3 = 0$ $\quad x = -\dfrac{3}{2}, \quad x = -\dfrac{1}{9}$

    (m) $3x^3 + 3x^2 - 6x = 0$ $\quad x = 0, \quad x = 1, x = -2$

    (n) $x^4 - 81 = 0$ $\quad x = 3, \quad x = -3$

    (o) $x^4 - 5x^2 + 4 = 0$ $\quad x = -1, \quad x = 1, \quad x = -2, \quad x = 2$

    (p) $x^4 + 10x^2 + 9 = 0$ No solution

    (q) $x^3 - 27 = 0$ $\quad x = 3$

    (r) $x^5 - 2x^3 + x^4 - 8x^2 - 8x + 16 = 0$ $\quad x = 2, \quad x = 1, \quad x = -2$

3. Solve the following equations:

    (a) $x^2 + 4x + 2 = 7$ $\quad x = 1, \quad x = -5$

    (b) $x^3 = 4x$ $\quad x = -2, \quad x = 0, \quad x = 2$

    (c) $x^2 + 8x + 6 = 3x$ $\quad x = -2, \quad x = -3$

(d) $2x(x + 11) = 13x + 5$ $\qquad x = -5, \quad x = \dfrac{1}{2}$

4. Find a polynomial equation that satisfies the given conditions. Both sides of the equation should be in Simplified Expanded Form.

   (a) has solutions $x = 1$, $x = 0$ and $x = -5$. $\qquad x^3 + 4x^2 - 5x = 0$

   (b) its only real solutions are $x = 3$, $x = \dfrac{3}{2}$ and has degree 3. $\qquad x^3 - \dfrac{15}{2}x^2 + 18x - \dfrac{27}{2}$

   (c) it has solutions $x = \dfrac{1}{2}$, $x = 2$, $x = -\dfrac{2}{3}$ and integer coefficients. $\qquad 6x^3 - 19x^2 + 16x - 4 = 0$