Review for Math 21, Fall 2009

Nikos Apostolakis

For the next three questions recall that while on Earth, having ten fingers, we use a base 10 system the inhabitants on planet Firfe, having only one hand, use a base 5 system and on planet Triton, where they have only one hand with three fingers, they use a base 3 system.

1. Complete the following table that translates numbers written in one of the three systems to the other two.

Earth	Firfe	Triton
123		
		201
	234	
100		
		100
	1234	
		1002
6561		
		11021202

2. Perform the following calculations as a Tritian would:

   (a)
   $$\begin{array}{r} 2101122112 \\ +\quad\ 121012 \\ \hline \end{array}$$

   (b)
   $$\begin{array}{r} 1320 \\ \times\quad 122 \\ \hline \end{array}$$

3. Perform the following calculations as a Firfian would:

   (a)
   $$\begin{array}{r} 123402 \\ +\quad 2314 \\ \hline \end{array}$$

   (b)
   $$\begin{array}{r} 3412 \\ \times\quad 403 \\ \hline \end{array}$$

4. Would you rather play first or second in each of the following Nim games?

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| | | | | | |        | | | | | | | | | | |
| | | | |            | | | | | | | | |
| | | |              | | | | | | |
| |                  | | | | | |
                     | | |
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5. If possible find a winning first move for each of the following Nim games:

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| | | | | | | | |      | | | | | | | | |      | | | | | | |
| | | | | | |          | | | | | |            | | | | |
| | | | |              | | | | |              | | |
| | |                  | | |                  |
```

6. 500 students were asked whether they enrolled in three classes, a math class, a literature class and a history class. 242 students took the math class, 278 took the literature class, and 298 took the history class. 43 students took math and literature, 50 literature and history, 75 all three classes and 17 students took none of these classes. How many students:

   (a) Took math and history?

   (b) Took only math?

   (c) Took took exactly two of the three classes?

   (d) Took exactly one of the three classes?

7. How many 7–digit numbers are there in Triton?

8. How many 4–digit numbers there are in Firfe? How many of those numbers have all different digits?

9. For the following question we look at only the five first letters of the alphabet:

   (a) How many five letter words can we make using only letters from the set $\{a, b, c, d, e\}$.

   (b) How many of those words start with a consonant?

   (c) How many of those words start with a vowel and end with a consonant?

   (d) How many of these words use all five letters?

   (e) Of all words in part (d) how many have the two vowels next to each other?

10. A license plate has the following format: A consonant letter from the English alphabet is followed by four non-zero digits and there is a vowel at the end. The letters W and Y cannot be used in these license plates. An example of one such license plate would be $Z1233A$. How many such license plates can be issued?

11. In how many ways can you permute the letters of the word BAD and generate new "words"?

12. In how many ways can you permute the letters of the word GOOD in order to generate new "words"? (Some examples: DOGO, OOGD, ODGO, etc.)

13. Three girls and three boys plan to go to a movie theater to see a romantic comedy. They want to sit on a row next to each other such that no two boys and no two girls are sitting next to other. In how many ways can they achieve that arrangement? What would be the number of seating possibilities, if one of the girls doesn't show up?

To answer the next three questions you will need to first figure out what terms like "full house" and "flush" mean. Wikipedia is your friend!

14. In how many ways we can get a full house in poker?

15. In how many ways we can get three of a kind in poker?

16. In how many ways we can get a flush in poker?

17. A *palindrome* is a word that reads the same when we read it backwards. For example the words "bob" and "tenet"are palindromes. If we are allowed to use only the letters a,b,c how many different palindromes of length 4 can we make? How many palindromes of length 5? or course, as usual, our words don't have to make sense in English.