

First Review for Math 06 Fall 2009

The answers

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Please note: You should fully justify your answers.

1. Perform the following operations. Simplify your answers as much as possible (but no more).

- (a) $\frac{5z - 12}{z^2 - 8z + 15} - \frac{3z - 2}{z - 3} + \frac{3}{z - 5}$
- (b) $\frac{2x^2 + x}{2x^2 - 5x - 3} \cdot \frac{x - 3}{x} \cdot \frac{x^2 - 9}{x^2 + 2x - 3}$
- (c) $\frac{\frac{ab}{a - b}}{\frac{b - a}{a}}$
- (d) $\frac{\frac{x}{x - 4} + \frac{1}{x + 2}}{\frac{4}{x - 4} - \frac{1}{x + 2}}$

Answer. (a) $-\frac{3z^2 + 25z - 31}{z^2 - 8z + 15}$

- (b) $\frac{x - 3}{x - 1}$
- (c) $\frac{2}{a^2 - b^2}$
- (d) $\frac{x - 1}{3}$

□

2. Solve:

- (a) $\frac{s}{2} - \frac{2}{s} = \frac{3s}{8}$
- (b) $\frac{x + 1}{x - 2} - \frac{x + 3}{x} = \frac{6}{x^2 - 2x}$
- (c) $\frac{3x}{x - 1} = \frac{2}{x - 2} - \frac{2}{x^2 - 3x + 2}$
- (d) $\frac{3x + 2}{3} = \frac{5x - 2}{5} + 4$
- (e) $\frac{x + 8}{x + 3} + 2 = -\frac{3x + 4}{10}$

Answer. (a) $s = 4$

- (b) All numbers are solutions except 0 and 2.
- (c) $\frac{2}{3}$
- (d) This equation has no solutions.
- (e) $x = -\frac{19}{3}$ or $x = -8$

□

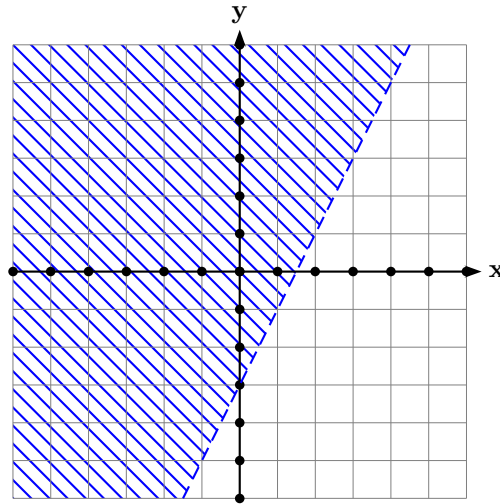
3. Determine the value of the real number a , if $x = 2$ is *not* a solution of the equation:

$$\frac{x - 2}{ax^2 + 3x - 10} = 0$$

Answer. $a = 1$

□

4. Write an inequality that describes the shaded region in the graph below:

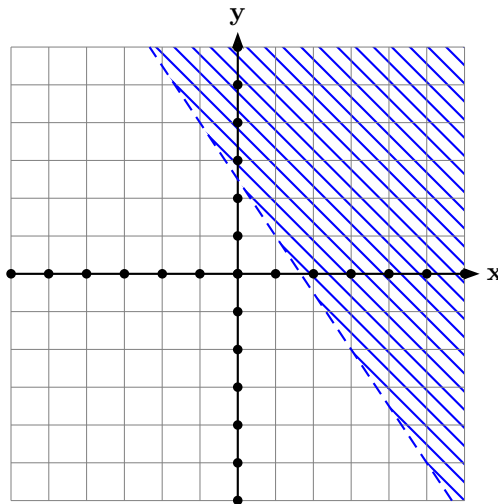


Answer. $y - 2x > -3$

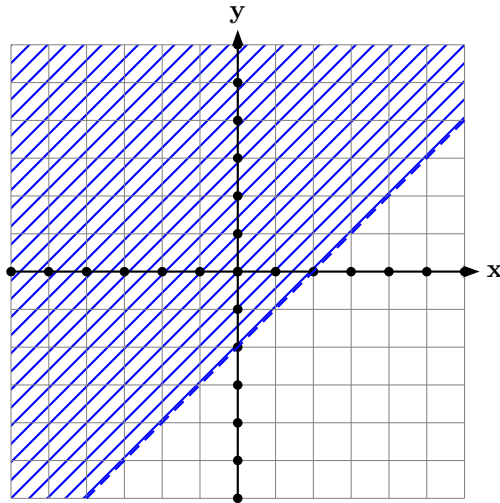
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5. Graph each of the following inequalities:

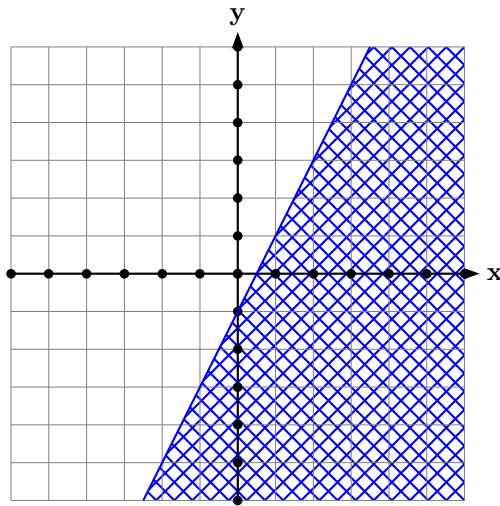
(a) $3x + 2y < 5$



(b) $-x + y > -2$

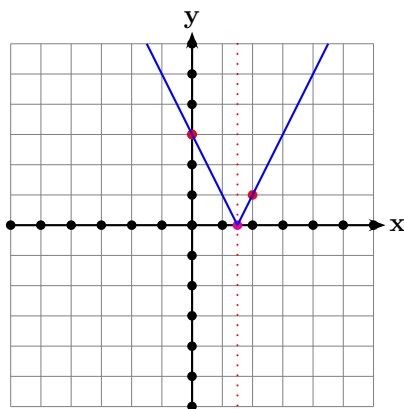


(c) $2x - y \geq 1$



6. Sketch the graph of $y = |2x - 3|$. Be sure to indicate clearly the critical point, the axis of symmetry and at least one point on either side of the axis of symmetry.

Answer. The critical point is at $(\frac{3}{2}, 0)$, and the axis of symmetry is $x = \frac{3}{2}$. To the left we have point $(0, 3)$ and to the right we have $(2, 1)$



□

7. Solve algebraically:

- (a) $|2x - 5| \geq 7$
- (b) $|2x + 6| < 8$
- (c) $|3x - 1| > -2$
- (d) $|4x - 31| \leq -1$

Answer. (a) $(-\infty, -1] \cup [6, \infty)$
(b) $(-7, 1)$
(c) All real numbers, i.e. $(-\infty, \infty)$
(d) There are no solutions, i.e. solution set is \emptyset

□

8. Simplify each of the following radical expressions:

- (a) $\sqrt{75} - 5\sqrt{12} + 3\sqrt{27}$
- (b) $(1 - \sqrt{3})(2 + \sqrt{6})$
- (c) $\frac{1 - \sqrt{10}}{\sqrt{6} + \sqrt{15}}$

Answer. (a) $4\sqrt{3}$
(b) $2 - 3\sqrt{2} - 2\sqrt{3} + \sqrt{6}$
(c) $\frac{\sqrt{15} - 2\sqrt{6}}{3}$

□

9. Evaluate

$$\frac{x^2 - y^2}{\sqrt{5}}$$

if $x = 1 + \sqrt{5}$ and $y = 1 - \sqrt{5}$

Answer. 4

□

10. Find b if $x = 1 + \sqrt{7}$ is a solution to the equation:

$$x^2 + bx - 6 = 0$$

Answer. $b = -2$

□

11. Solve:

$$\sqrt{15 - 2x} = x$$

Answer. $x = 3$

□

12. Solve:

$$\sqrt{7x + 1} = 3x + 1$$

Answer. $x = 0$ or $x = \frac{1}{9}$

□

13. Solve:

$$\sqrt{30 - 10x} + 3 = x$$

Answer. $x = 3$

□

14. Solve:

$$\sqrt{x+3} - \sqrt{3x+7} = -2$$

Answer. $x = 6$

□

15. Solve:

$$\sqrt{\sqrt{x+1} + 1} = \sqrt{x-4}$$

Answer. $x = 8$

□

16. Perform the following operations. Give your answer in the form $a + bi$ with a and b real numbers.

(a) $(2 - 3i)(-5 + 4i)$

(b) $\frac{(3 - 4i)(-1 + 2i)}{2 - i}$

(c) $\frac{-1 - i}{2 - 3i}$

(d) $(2 - i)^3$

Answer. (a) $2 + 23i$

(b) $5i$

(c) $\frac{1}{13} - \frac{5}{13}i$

(d) $2 - 11i$

□

17. Evaluate each of the following expressions when $z = -2 + 3i$:

(a) $z^2 + 4z$

(b) $\frac{3 + 2i}{iz}$

Answer. (a) -13

(b) -1

□

18. Solve the following equations using the quadratic formula:

(a) $x^2 + 8x + 15 = 0$

(b) $x^2 - 4x + 7 = 0$

(c) $x^2 + 17 = 0$

(d) $x^2 - 3x = 5$

(e) $2x^2 - 11x + 15 = 0$

(f) $3x^2 + 5x - 12 = 0$

(g) $10x^2 - 17x = -33$

Answer. (a) $x = -3$ or $x = -5$

- (b) $x = 2 \pm i\sqrt{3}$
- (c) $x = \pm i\sqrt{17}$
- (d) $x = \frac{3 \pm \sqrt{29}}{2}$
- (e) $x = 3$ or $x = \frac{5}{2}$
- (f) $x = -3$ or $x = \frac{4}{3}$
- (g) $x = \frac{17 \pm i\sqrt{1031}}{20}$

□

19. Factor each of the following quadratic trinomials using the quadratic formula:

- (a) $x^2 - 4$
- (b) $2x^2 + 50$
- (c) $7x^2 - 3x$
- (d) $x^2 - 5x + 6$
- (e) $6x^2 + x - 1$
- (f) $x^2 + 2x - 2$
- (g) $x^2 - 4x + 1$
- (h) $4x^2 - 8x + 13$

Answer. (a) $(x + 2)(x - 2)$

- (b) $2(x + 5i)(x - 5i)$
- (c) $x(7x - 3)$
- (d) $(x - 2)(x - 3)$
- (e) $(2x + 1)(3x - 1)$
- (f) $(x + 1 + \sqrt{3})(x + 1 - \sqrt{3})$
- (g) $(x - 2 + \sqrt{3})(x - 2 - \sqrt{3})$
- (h) $(2x - 2 - 3i)(2x - 2 + 3i)$

□

20. Find a quadratic equation with real coefficients that has $2 + 3i$ and $2 - 3i$ as solutions.

Answer. $x^2 - 4x + 13$

□

21. Can you find a quadratic equation with real coefficients that has $1 - i$ and $2 + i$ as solutions?

Answer. No.

□

22. The quadratic equation $x^2 + bx + 5 = 0$ has a double solution. Find b .

Answer. $b = 2\sqrt{5}$ or $b = -2\sqrt{5}$

□