Third Quiz for CSI35

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October 17, 2009

Directions: This quiz is due Tuesday October 20, at 4:00 PM. Please *staple* all the papers of your answer together.

1. Give definitions of:
   (a) Reflexive zero–one matrix.
   (b) Irreflexive zero–one matrix.
   (c) Symmetric zero–one matrix.
2. Give definitions of:
   (a) Reflexive digraph.
   (b) Irreflexive digraph.
   (c) Symmetric digraph.
3. Show that under the basic correspondence between digraphs and zero–one matrices:
   (a) Reflexive matrices correspond to reflexive digraphs.
   (b) Irreflexive matrices correspond to irreflexive digraphs.
   (c) Symmetric matrices correspond to symmetric digraphs.
4. A $n \times n$ zero-one matrix A is called *antisymmetric*, if and only if, for all indices i, j with $1 \leq i, j \leq n$ we have:
   $$a_{ij} = 1 \wedge a_{ji} = 1 \Longrightarrow i = j$$
   A $n \times n$ zero-one matrix A is called *asymmetric*, if and only if, for all indices i, j with $1 \leq i, j \leq n$ we have:
   $$a_{ij} = 1 \Longrightarrow a_{ji} = 0$$
   (a) Are all antisymmetric matrices asymmetric? Prove your answer.
   (b) Are all asymmetric matrices antisymmetric? Prove your answer.
   (c) Give the corresponding definitions of asymmetric and antisymmetric digraphs.

5. (a) How many reflexive $n \times n$ zero–one matrices are there?
   (b) How many irreflexive $n \times n$ zero–one matrices are there?
   (c) How many symmetric $n \times n$ zero–one matrices are there?
   (d) How many antisymmetric $n \times n$ zero–one matrices are there?
   (e) How many asymmetric $n \times n$ zero–one matrices are there?
   (f) How many reflexive and symmetric $n \times n$ zero–one matrices are there?
   (g) How many reflexive and antisymmetric $n \times n$ zero–one matrices are there?
   (h) How many reflexive and asymmetric $n \times n$ zero–one matrices are there?

6. Find the digraph $G_2 \circ G_1$ if the digraph G_i corresponds to the matrix M_i for $i = 1, 2$.

$$M_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

7. Draw a digraph G as follows: There is one vertex for each number in the set $\{1, 2, \ldots, 20\}$, and there is an edge with source the vertex m and destination the vertex n if and only if m and n leave the same remainder when divided by 5. Is the graph G
   (a) reflexive?
   (b) symmetric?
   (c) antisymmetric?

8. Draw a digraph H as follows: There is one vertex for each element of the set $\{2, 3, ..., 10\}$, and there is an edge with source the vertex m and destination the vertex n if and only if m divides n. Is the graph H
   (a) reflexive?
   (b) symmetric?
   (c) antisymmetric?