First Quiz for CSI35

Nikos Apostolakis

September 5, 2008

Directions: This quiz is due Thursday September 11, at 2:00 PM.

1. Prove that for all natural numbers $n \geq 1$ we have:

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Prove that for all $n \in \mathbb{N}$, $n^2 + n$ is even.

3. Prove that for all $n \in \mathbb{N}$, 6 divides $n^3 - n$.

4. Alice and Bob play a game by taking turns removing 1, 2 or 3 stones from a pile that initially has n stones. The person that removes the last stone wins the game. Alice plays always first.

   (a) Prove by induction that if n is a multiple of 4 then Bob has a wining strategy.

   (b) Prove that if n is not a multiple of 4 then Alice has a wining strategy.

5. In the Country of Oz they have only 3–cent and 5–cent postage stamps. Prove that you can use these stamps to pay for any letter that costs 8 or more cents.

6. **Extra Credit:** Can you figure out what is wrong in the following "proof"?

   Claim. All cars have the same color.

   Proof. There is only a finite number of cars in the world, let's say n. I will prove, using mathematical induction, that for any natural number $n \geq 1$, in any collection of n cars all cars have to have the same color.

   Base step: For $n = 1$ the proposition is obvious.

   Inductive step: Assume that the cars in any collections of n cars have the same color. I will prove that the cars in any collection of $n+1$ cars have the same color. Indeed, let $\{c_1, c_2, \ldots, c_n, c_{n+1}\}$ be a collection of $n+1$ cars. By the inductive hypothesis, the cars $c_1, c_2, \ldots, c_n$ have the same color so I need to show that the car c_{n+1} has the same color as well. But this is true because by the inductive hypothesis the cars $c_2, \ldots, c_n, c_{n+1}$ have the same color. □