Practicing Mathematical Induction
Handout for **CSI35**

1. Prove by induction that for all $n \in \mathbb{N}$:

$$2 + 4 + \cdots + 2n = n^2 + n$$

Answer. (a) First write proposition we want to prove in a consise form:

$$\sum_{k=} \quad =$$

(b) **Base Step** What does the proposition say for $n = 0$?

Verify this statemet:

(c) **Inductive Step** Now we assume that the proposition is proved for some natural number n and (using this assumption) we will prove it for $n+1$. So we assume that

and we want to prove that:

Now prove it:

□

2. Prove that for all natural numbers $n \geq 1$ the following identity is true:

$$(1-x)(1+x+x^2+\cdots+x^n) = 1-x^{n+1}$$

Answer. (a) **Base Step:**

(b) **Inductive Step:**

□

3. Prove that for all natural numbers $n \geq 4$ the following inequality holds:

$$2^n < n!$$

Answer. (a) **Base Step** We must check the statement for $n =$:

(b) **Inductive Step** Now we assume that the proposition is proved for some natural number n and (using this assumption) we will prove it for $n+1$. So we assume that

and we want to prove that:

Now prove it:

□

4. When is $5^n < n!$? Make a conjecture and then prove it inductively.

5. Prove that for any positive integer n, 5 divides $n^5 - n$