# Take home exam for CSI35

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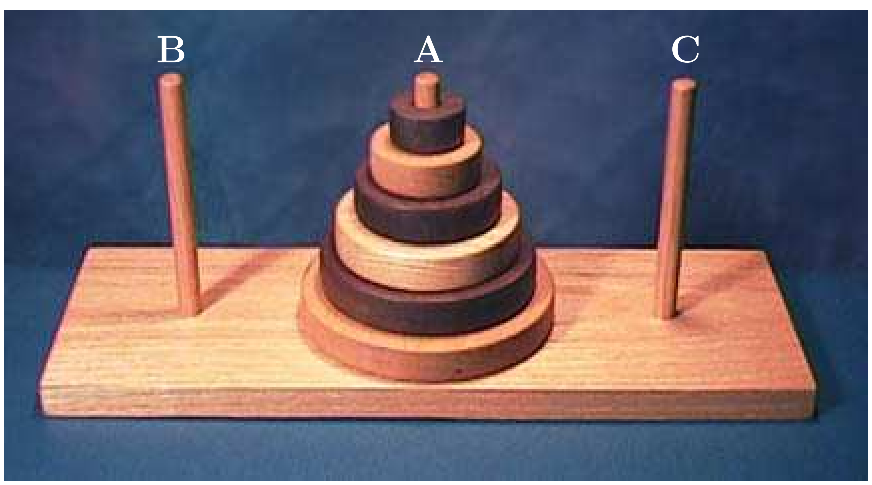
October 23, 2008

**Directions:** Please write your answers in separate papers and staple all the papers together. This quiz is due Tuesday October 28, at 2:00 PM.

1. Give a recursive definition of the sequence $\{a_n\}$ if

$$a_n = 5n - 6, \quad \text{for} \quad n = 0, 1, 2, \ldots$$

2. This question is about the Tower of Hanoi puzzle. The puzzle consists of three pegs and six disks, initially stacked in decreasing size on one of the pegs, say $A$, as shown below:




The goal is to transfer the whole tower of six disks to one of the other pegs, say $C$, moving only one disk at a time and never moving a larger one on top of a smaller.

(a) Find a solution to the puzzle.

(b) Prove that this puzzle has a solution for any number of initial disks.

(c) Prove that the puzzle with $n$ disks can be solved in $2^n - 1$ moves.

(d) Prove that the puzzle with $n$ disks cannot be solved in fewer than $2^n - 1$ moves.

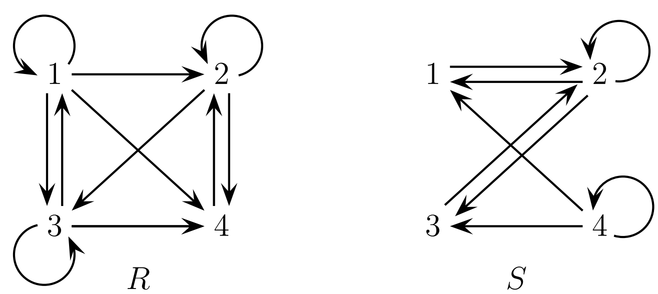
3. If $w$ is a string then its *reverse* $w^R$ is the string obtained by reading $w$ backwards, for example the reverse of the string *sub* is the string *bus*.

   (a) Give an inductive definition of the reverse of a string.

   (b) Use structural induction to prove that for all strings $w_1$and $w_2$ the following holds:

$$(w_1w_2)^R = w_2^R w_1^R$$

4. Consider the relations $R$, and $S$ on the set $\{1, 2, 3, 4\}$ represented by the digraphs:




   (a) Find the matrices $M_S$ and $M_R$.

   (b) Use these matrices to compute the compositions $R \circ S$ and $S \circ R$.

   (c) Draw the digraphs that represent $R \circ S$ and $S \circ R$.

5. Consider the relation $R$ represented by the matrix

$$M_R = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Is $R$ reflexive?

   (b) Is $R$ symmetric?

   (c) Is $R$ transitive?

   (d) Draw the digraph representing $R$.

6. Which of the following relations defined on the set of all people are equivalence relations. Justify your answers:

   (a) $(a, b) \in R$ iff $a$ has the same parents as $b$.

   (b) $(a, b) \in R$ iff $a$ is parent of $b$.

   (c) $(a, b) \in R$ iff $a$ lives in the same town as $b$.

   (d) $(a, b) \in R$ iff $a$ lives one floor above $b$.

   (e) $(a, b) \in R$ iff $a$ is an acquaintance of $b$.

7. Consider the relation $R$ defined on the set of all positive real numbers as follows:

$$(a, b) \in R \quad \text{iff} \quad \frac{a}{b} \in \mathbb{Q}\,,$$

where $\mathbb{Q}$ stands for the set of rational numbers. Prove that $R$ is an equivalence relation.

8. (**Extra Credit**) List all equivalence relations on the set $A = \{1, 2, 3, 4\}$.